AMERICA'S CHOICE®

SAVVAS

RESEARCH BASIS

MATHEMATICS NAVIGATOR

In 1970, the United States produced more than half (50%) of the world's science and engineering doctorates; by 2010, our share is projected to fall to about 15%.

-Richard Freeman, National Bureau of Economic Research (U.S. Dept. Ed. 2006)

MATHEMATICS NAVIGATOR

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Background

In the Trends in International Math and Science Study (TIMSS), 7% percent of both 4th- and 8th-grade students performed at the "advanced" level, the highest international benchmark (Gonzales et al., 2004). By comparison, 38% of 4th- and 8th-grade students from Singapore reached the same advanced benchmark (Ministry of Education, 2004). From 1995 to 2003, U.S. 4th graders showed no measurable change in their mathematics performance. Although 8th-grade students did see some improvement, with higher average scores in 2003, this increase "did not keep pace with improved scores among students in several other countries" (Gonzales et al., 2004). The United States has a deficit in mathematics knowledge compared to other countries in school mathematics. As this deficit grows, it becomes more difficult to overcome, and the probability of the United States regaining global competitiveness in mathematics diminishes.

This problem is not reserved to the early grades; it manifests in higher-educational settings as well. Remedial courses are not offered in all colleges, but a surprising number of students enroll in them where available. Statistics show that 63% of freshmen in 2-year public institutions took remedial courses and 38% of freshmen in 4-year public institutions did the same. Unfortunately, research also shows that students who require remedial work at the college level are less likely to earn a degree or certificate than those who do not—about 50% less likely (Vail, 2004; NCES, 2004).

Many of our math students are falling further and further behind, and our nation could soon experience a crisis as a result. If this trend continues, there will not be enough young talent necessary to produce the scientists, researchers, mathematicians, and engineers that our country requires to remain globally competitive and domestically sound in the coming years. It is no longer enough to improve; our students must accelerate to keep pace, and the bar must be set higher.

Intervention is needed now. With a series of American Competitiveness Initiatives, the federal government is pushing to improve math education in the elementary and middle grades. Pursuant to this goal, the National Mathematics Advisory Panel was formed to make recommendations geared to "promote scientifically-based practices in math instruction so students are prepared for success in algebra and more advanced math in middle and high school" (U.S. Dept. of Ed., 2006). The National Mathematics Advisory Panel is chaired by Dr. Larry Faulkner and seated with several expert panelists, including Liping Ma, Deborah Ball, James Stigler, and Hung-Hsi Wu, whose research, quoted herein, forms the basis of Mathematics Navigator.

Introduction to Mathematics Navigator

Mathematics Navigator is a highly flexible, research-based program for elementary, middle, and early high school. It is designed to address the needs of struggling students who are currently working at grade level, but are having difficulty keeping up with the rest of the class. At some point in their academic careers, these students may have received weak preparation in certain concepts, or they may have misunderstood the concepts altogether. These specific misconceptions and gaps in mathematical understanding only frustrate these students' attempts to succeed as they encounter more advanced mathematics.

The Navigator intervention does not revisit or repeat initial teaching. Rather, unlike many other interventions, the program focuses on revising misconceptions. Reteaching does not necessarily help students identify and expose misunderstandings; it simply repeats instruction that didn't work for students the first time around. Navigator is not remedial in nature; instead, it lays out a program that instructors can use in whole or in part to help students reconstruct their conceptual framework. The accompanying instructor materials provide explicit discussion of the mathematics, detailed symptoms that signal the presence of misconceptions, student work samples illustrating misconceptions, and numerous ideas for addressing problems.

Because academic success can hinge on seemingly minor gaps in mathematical understanding, Navigator capitalizes on students' prior knowledge. The program is designed to help instructors and students seek out faulty knowledge and move through a process to revise and repair the faults. The unique nature of Navigator tasks, along with the established rituals for engaging in the tasks, encourage students to approach the mathematics from several angles. Students do much more than solve problems; they analyze their work for errors, test the validity of their work against other students' work, try multiple strategies to determine if their prior knowledge holds true or fails them, and modify or invent similar problems. All this leads to revising misconceptions. In the process, students learn to problem-solve, reason mathematically, and justify their work.

America's Choice has assembled a discrete set of mathematics concepts into 18 stand-alone modules of study. Each module contains a logical progression of 20 sessions that are carefully sequenced to deepen students' conceptual understanding. Each session focuses on specific points in students' mathematical understanding, and provides tasks and problems that are deliberately designed to uncover misconceptions. Careful scaffolding requires students to probe more deeply into the mathematics with each successive task. Navigator offers short-term, targeted intervention meant to augment, *not replace*, existing curriculum. The program does not disrupt existing schedules because the half-hour to 45-minute sessions take place outside regular math class.

Mathematics Navigator is based on the results of over a decade's worth of research, as well as field-tests (America's Choice, 2006). America's Choice believes that with extra time, effort, and instruction, *all* students have the ability to reach high standards—*even in math*. By making students active participants in their learning process and fostering a supportive and innovative learning environment, Mathematics Navigator can help struggling students get back on track for success in higher-level mathematics.

Introduction to the Six Main Premises

1. Targeted Concepts

Malcolm Swan (2005) believes that if students are to gain a deep understanding of mathematical concepts, they must study "an interconnected network of interesting and powerful ideas to actively explore, debate and gradually come to understand." Alan Ginsburg et al. (2005) argue for "developing a rich mathematical treatment of each topic." Malcolm Swan (2005) states that "there is a vast body of research literature documenting learners' mistakes in mathematics. This work shows that mistakes are often the result of consistent, alternative interpretations of mathematical ideas." Because these misconceptions are consistent, the concepts can be targeted.

2. The Algebraic Structure of Arithmetic

Quite often, early instruction in arithmetic and mathematics fails to take into account the goal of algebra preparedness. According to researchers such as Liping Ma (1999), even concepts in early arithmetic, such as addition and division, can be presented and developed in ways that lead to easier transitions to algebra. Teacher education materials are beginning to emphasize the algebraic structure of arithmetic. The Conference Board of the Mathematical Sciences suggests that "although the study of algebra and functions generally begins at the upper-, middle-, or high-school levels, some core concepts and practices are accessible much earlier" (Tucker et al., 2001).

3. Prior Knowledge and Misconceptions

Alan Bell (2005) argues that without "exposure of pupils' misconceptions and their resolution through conflict discussion, students may not know why a mistake occurred." Mathematics interventions should use a subtle process to expose flawed thinking and allow students to confront their own misconceptions and, consequently, discover for themselves the source of their mistakes (Bell, 2005; Bell and Swan 2006; Donovon and Bransford, 2005). In order to engage students in misconceptions, students must be "active rather than passive learners" (Swan, 2005).

4. The Language-Rich Environment

Building technical vocabulary is an important yet often neglected part of mathematics instruction. Students should be able to discuss mathematics using the academic language of the discipline and talk that is accountable. "Accountable talk...puts forth and demands knowledge that is accurate and relevant to the issue under discussion... uses evidence appropriate to the discipline...and follows established norms of good reasoning" (Resnick, 1999). In a similar vein, Lily Wong Fillmore and Catherine Snow (2002) suggest that "For the most part, academic English is learned over the course of schooling through frequent engagement in classroom talk, reading textbooks, and writing."

5. Better Learners of Mathematics

Lev Vygotsky's work shows that learning is directly influenced by the social and cultural contexts in which it takes place. He illustrates the fact that knowledge gains that normally cannot be achieved alone become possible when learning takes place alongside a more capable peer or adult (Vygotsky, 1962, 1978). This view of learning is similar in some respects to the centuries-old practice of apprenticeship that, according to Lauren Resnick, allows learners "to acquire the complex interdisciplinary knowledge, practical abilities, and appropriate forms of social behavior that went with high levels of skilled performance" (Resnick & Nelson-Le Gall, 1997). Similarly, researchers from Project Zero state that "Even though in U.S. culture we tend to separate the *how* from the *what* of learning, we think the two are integrally linked. We suggest that the focus...should be making learning visible...both the acts and products of learning..." (Project Zero, 2006).

6. Instructional Support

"Ideally, support for teachers' learning of the instructional design and teaching practices would come from the materials of the program..." (Hill and Ball, 2004). In a 2005 issue of *Essential Information for Education Policy*, the American Education Research Association stated that "teacher professional development can improve student achievement when it focuses on teachers' knowledge of the subject matter and how students understand and learn it" (AERA, 2005). Additionally, the California State Department of Education concluded in 2006 that "Students with special needs must have access to the same academic standards-based curriculum provided to all students. Instructional materials must be designed to help meet the needs of those students."

Research Details for the Six Main Premises

1. Targeted Concepts

What the Research Says

The 2003 TIMSS results revealed that Singapore, Hong Kong SAR, Japan, and Chinese Taipei students scored the highest in math. When comparing the curriculum used in the most successful programs with the American curriculum, some major differences become apparent. In the top-performing countries, mathematics consists of a deep incisive study of the conceptual framework of mathematics. The materials contain fewer but richer problem situations that enable students to manipulate and probe the mathematics from every angle.

There is a considerable body of research documenting students' mistakes in mathematics. (For example the reports of the Assessment of Performance Unit (Foxman et al., 1980), the Concepts in Secondary Mathematics and Science Project (Hart, 1980, 1981), research reviews (e.g., Dickson et al., 1984; Bell et al., 1983) and the more recent work based on children's answers to National Curriculum Test questions, (e.g., Ryan and Williams, 2000). This work reveals that there are a limited number of identifiable misconceptions. Malcolm Swan (2005) states that, "Such misconceptions should not be dismissed as 'wrong thinking' as they may be necessary stages of development." For example, most learners generalize from their early experiences that:

- you can't divide smaller numbers by larger ones
- division always makes numbers smaller
- the more digits a number has, then the larger is its value
- shapes with bigger areas have bigger perimeters
- letters represent particular numbers
- "equals" means "makes" (Swan, 2005)

Interventions can be targeted at these misconceptions and their related concepts.

What Navigator Does

Navigator engages students in a *deeper study of fewer, more critical concepts*. Navigator requires students to interact with the mathematics, while *balancing work in concepts, problem solving, and skills*.

Deeper Study of Fewer, More Critical Concepts

Navigator sessions are structured around a handful of tasks that students study deeply. By including only a few problems, Navigator provides students with extra time to rework their computations or try other methods. Most importantly, however, the inclusion of fewer tasks gives students time to think and form questions. For example, students learn a complete process for solving word problems: they consider the problem and make an estimate; show all work; represent the mathematics in an equation; use a number line, chart, or other graphic to illustrate the mathematics; and write an answer to the problem in a complete sentence. Students will repeatedly hear, see, and use strategies such as these as they discuss critical mathematical concepts and share problem-solving strategies with partners.

Navigator focuses on conceptual depth rather than procedural finesse. It is this conceptual depth that students who seem to grapple with mathematics in vain are lacking. Struggling students invariably do not have the dependable metacognitive strategies that they need in order to approach math problems. These students in particular need to routinely self-monitor their work, but they often lack the skills to do so. Navigator helps students build these metacognitive strategies and teaches them to reason mathematically, while focusing on a select group of core concepts.

Balance of Work in Concepts, Problem-Solving, and Skills

Students who are performing unsatisfactorily need to regain control of the concepts—they need explicit instruction and opportunities to demystify their confused interpretations of concepts. Critical thinking and problem solving must be taught over time, practiced daily, and shared with others. Navigator provides an evenly balanced focus on concepts, problem solving, and skills. Concepts are studied and connected so that students can see how these concepts apply to a range of mathematical situations. Problem-solving strategies are routinely shared with partners to deepen students' perspectives and broaden their array of possible solution paths. Procedural and computational skills are a necessary component to a students' mathematical repertoire. In Elementary Mathematics Navigator, daily sessions begin with 6 minutes of skills practice that emphasizes patterns and promotes strategies for successful computation. Middle School Navigator has skills practice every fifth day.

2. The Algebraic Structure of Arithmetic

What the Research Says

In order to progress to more advanced mathematics, students must establish a deep and unambiguous hold on a coherent set of concepts—concepts that lead directly to the fundamentals of algebra. In the most successful countries, mathematics instruction emphasizes the algebraic structure of arithmetic, even (and especially) in early elementary instruction. Liping Ma and others suggest that early concepts in arithmetic can be developed in a manner that builds layers of understanding and allows for more elegant transitions to advanced mathematics. For example, less than a handful of basic ideas—ideas that are routinely and readily learned by students in other countries—are needed to highlight the algebraic side of adding and dividing. If mastery of these ideas were the focus in U.S. curriculum, our students would have a much easier time when they graduated to work with fractions (Ma, 1999; Wu, 2002). Lesley Booth's research underscores the benefit of connecting arithmetic to algebra and enlisting active engagement of students' reasoning abilities early on. "In arithmetic, the focus of activity is the finding of particular numerical answers. In algebra, however, this is not so. In algebra, the focus is on the derivation of procedures and relationships and the expression of these in general, simplified form... Many students do not realize this; they assume that what is required is a numerical answer" (Booth, 1999).

The research of Thomas Carpenter and others provides a simple yet profound example of the disconnected nature of arithmetic and algebra in our curriculum. Their studies show that many U.S. students develop misconceptions about the "equals" (=) sign. It is probably unfair to expect otherwise, since it is nearly impossible for students to posses complete understanding of a mathematical symbol whose use in early arithmetic offers no connection to its true meaning or to its later use in algebra. American students become convinced that the = sign is a signal to "do something" or "to compute," something like punching the button on a simple adding machine. They altogether miss the idea that = is a mathematical symbol denoting the concept of equivalency (Carpenter, et al., 2003). As Suzanne Chapin et al. (2003) succinctly state, "Activities and discussions that revolve around patterns, equality, properties, generalizations, and symbolism are the precursors to the formal study of algebra."

Researchers have found that young students have implicit knowledge of properties in mathematics. Many students recognize that numbers in a simple addition problem are interchangeable, e.g., that 5 + 2 is equivalent to 2 + 5. Yet if this simple number property has not been taught explicitly, many students do not make the connection to other contexts. For instance, they may not understand that the commutative property of numbers applies to more "complicated math" like larger numbers or fractions. Researchers are finding "that elementary students can learn to think about arithmetic in ways that both enhance their early learning of arithmetic and provide a foundation for learning algebra" (Wisconsin Center for Education Research, 2000).

What Navigator Does

With Navigator, students learn mathematics in a way that familiarizes them with the algebraic structure of mathematics. Navigator prepares students for more advanced mathematics while incorporating the most *elementary aspects of algebra*.

Elementary Aspects of Algebra

Navigator focuses on the most fundamental aspects of algebra, such as horizontal notation or graphical representations, in all of its modules. Algebra is the fundamental language of math, yet most U.S. mathematics courses hardly hint at algebraic structure. Navigator connects arithmetic to algebraic structures and widens the scope of students' mathematical understanding, making for easier transitions to algebra.

Simple adjustments in the early presentation of arithmetic can create a fundamental cognitive shift for students. For example, when students are asked to add 13 and 5 simply by "lining the numbers up on the right," the concept of place value is left ambiguous.

$$\begin{array}{r} 13 \\ \underline{+5} \end{array}$$

However, if a horizontal format is also introduced, students become familiar with the notation used with algebraic equations.

13 + 5 = __   5 + __ = 8   __ + 5 = 8

And then, some time later, it is no great leap for students to grasp the essential algebraic concept of using letters to represent values.

$3 + 5 = y$ $5 + x = 8$ $x + 5 = 8$

The ultimate goal of Navigator is to get students back on track and progressing toward advanced mathematics. This goal prompts a single, simple question: What will students need when they get there? The answer: A solid understanding of essential concepts that transfer readily to algebra—precisely what Navigator delivers.

3. Prior Knowledge and Misconceptions

What the Research Says

Students bring prior knowledge of mathematical concepts to class, and research shows that working with that prior knowledge can lead to deeper understanding and long-term learning (Askew, 2002). Malcolm Swan places great importance on directing students' prior knowledge to problems that have been carefully devised to produce incorrect responses. Wrong answers often reveal contradictions in the way students have been perceiving the target concept. "Research has shown that teaching becomes more effective when common mistakes and misconceptions are systematically exposed" (Swan, 2005). Swan's strategies develop collaborative, creative, and active learners whose task is to discover the source of their own misconceptions. The work focuses on math activity rather than math answers. Swan comments, "...we do not concern ourselves with whether or not learners complete every task, but instead we try to increase their power to explain and use mathematical ideas" (Swan, 2005). Alan Bell suggests that when students face a challenge to their cognitive structure, they are much more willing to stretch themselves intellectually. Conceptual gains realized in this manner promote "transfer from the immediate topic to wider situations" (Bell, 2006).

According to Jack Lochhead and Jose Mestre (1988), students hold on tenaciously to ill-conceived mathematical ideas because these are the ideas they have believed in for years. Lochhead and Mestre suggest that students "...must actively participate in the process of overcoming their misconceptions." Suzanne Chapin points out that students' math talk "about solution methods and problem solving strategies can reveal shallow understanding or holes in previously learned concepts, as well as misconceptions or over-generalizations" (Chapin et al., 2003). To uncover their misconceptions, students must analyze their work carefully, consider the mathematics from different perspectives, compare solutions, develop alternate methods, and invent similar problems. Chapin and co-authors describe a similarly engaging and pro-active form of student learning in which the focus is on "... processing information, applying reasoning, hearing ideas from others, and connecting new thinking to what they already know, all for the goal of making sense for themselves of new concepts and skills." When it comes to revising long-held or stubbornly-held misconceptions, Chapin hits the nail on the head: "The source of the knowledge, of creating new understanding, lies within the student, and making sense is the key" (Chapin, et al., 2003).

Malcolm Swan (2005), in *Improving Learning in Mathematics: Challenges and Strategies*, identifies learning opportunities that elicit prior knowledge, uncover misconceptions, and engage students in the concepts. Swan maintains that instruction is off-track when students feel mathematics is something that is "done to them," rather than a creative, stimulating subject to explore. When students are given low-level tasks without challenge, and when they listen to instruction but are not able to participate in learning by discussing or explaining their understanding, the exercise becomes nothing more than a simple transmission session.

What Navigator Does

Mathematics Navigator places an emphasis on prior knowledge and *revising misconceptions*. Prior knowledge is activated in a way that pulls students out of passive modes of learning and spurs them on to an *active and determined study of the concepts*.

Revising Misconceptions

The Navigator program is based on the premise that students' faulty prior knowledge can be harnessed and used to their benefit. Navigator instructors are taught to treat students' misconceptions as the key to helping students revise their knowledge.

Navigator tasks are deliberately designed to uncover student misconceptions and illuminate errors caused by faulty reasoning. When flawed thinking manifests itself in students' answers, this thinking can be brought into the light. A dynamic tension arises between what students expect to accomplish through the application of their misconception and what actually occurs. When students realize their prior knowledge has failed them, they are motivated to understand why. This cognitive dissonance compels students to figure and re-figure the mathematics, compare solutions, try alternate strategies, and work backward through the math until they discover the source of the error. Such processes lead students well beyond the task of getting the answers and into the realm of deeper understanding.

Active and Determined Study of the Concepts

In a Navigator classroom, it is the process of *doing* the mathematics that is important, not the number of problems answered, or answered correctly. America's Choice wants to see students shift their focus from getting answers to understanding the mathematics. That is why Navigator is highly collaborative and verbal. The Navigator program uses expertly-crafted problems and tasks that elicit active engagement focused on misconceptions. When students come face to face with their incomplete or ill-conceived ideas about how mathematics works, they cannot help but delve in, root around, and talk about why and how their reasoning failed. The right kind of classroom environment can make an isolated student an active learner. Navigator provides such an environment, in which students collaborate with partners, share problem-solving strategies, discuss the concepts, explain their work, and begin developing justifications. They challenge one another with invented problems, write about their new understanding, and take part in whole-group discussions and group presentations. Students do all this while using and developing their academic vocabulary by using the terms and definitions of the mathematics discipline.

4. The Language-Rich Environment

What the Research Says

From an instructional perspective, one needs to be aware that language and mathematical competence are intertwined and therefore must be taught collectively if students are to make strong academic progress in mathematics (Cummins, 1984). Hilary Shuard and Andrew Rothery warn against the casual disregard of academic language. "There is a danger of pupils not having the language experience and the clear explanations of mathematical words which they need to support their reading of the text… This problem may be overcome by a more explicit approach to technical vocabulary in arithmetic lessons..." (Shuard and Rothery, 1984).

Suzanne Chapin argues that talking and interacting with other students around the mathematics is an important part of the learning process. "Talk can be used to assist students in organizing what they already know into larger and more powerful conceptual structures—the big ideas of mathematics… Discussing the concepts on which procedures are based as well as the reasons behind the steps in any procedure can serve to strengthen students' understanding of both" (Chapin et. al., 2003).

For English language learners (ELL), social language develops more quickly than academic language, because they have more exposure to it. Academic language is used less often and developed more slowly. Once the set of academic vocabulary and technical descriptions are acquired, however, it is a much easier language set to manage because it is free of the nuances and cultural adaptations found in social language. Catherine Snow summarizes this idea as follows:

> *The vocabulary of academic language goes well beyond that used in most social conversations. It is only through structured talk about academically relevant content that students will learn the words needed to engage in class discussions and to comprehend what they read in various subjects. Memorizing word lists rarely works. Words must be learned and used in context.… What is important is that teachers have deliberate strategies for clarifying word meanings and that children have opportunities to use those words in context.* (Snow, American Educational Research Association, 2004)

Classroom talk is pivotal to growing knowledge and refining understanding. "Carefully guided classroom talk is an especially effective method for developing concepts and building connections among mathematical ideas" (Chapin et al., 2003). Researcher Hermine Marshall (2001) describes a learning-oriented classroom in which the "work is purposeful and directed at making personalized and collective sense through a building up of connections, applications, and associations." An important aspect of this learning-oriented classroom is its behavior as a community that considers the generation of collective knowledge equally important to individual knowledge. As Project Zero (2006) states, "We suggest that the focus of documentation should be making learning visible... both the acts and products of learning, as well as supporting and nurturing that learning."

What Navigator Does

Students *collaborate and talk about mathematics* in a language-rich environment to deepen and accelerate their learning. Then they refine their understanding and make *new knowledge gains visible* to others.

Collaborate and Talk about Mathematics

Collaboration is key to the Navigator experience, and discussion is a vital component. Navigator classrooms are language-rich, not language-free. Students must routinely articulate their understanding of mathematical situations; in this way academic language is introduced and explicitly defined. It is then used over and over again in the context of the mathematics. The development of increasingly sophisticated academic language must occur alongside the development of students' mathematical abilities. Learning the proper vocabulary of any discipline is critical to gaining knowledge in that field.

The highly-verbal classroom is particularly ideal for moderately-skilled English language learners. Their initial discussions are often with same-language speakers, but students return to whole-group discussions by the end of class. Navigator helps all students build their collaborative skills while perfecting problem solving. Except for brief skills practice and solo work, Navigator sessions are almost entirely composed of collaborative efforts to deepen math understanding.

Make New Knowledge Gains Visible

The most valuable product that students put forth in a Navigator session is not a list of answers, a sheet of flawless skill drills, or the solution to a particularly difficult word problem, but rather visible, audible, or readable representations of student's knowledge gains and revised misconceptions. Students work with partners every day to check each other and identify mistakes, and then share their work and ideas in whole-class discussions. In this way new knowledge is made visible to all students. In addition, students, including ELL students, are encouraged to use pictures, diagrams, and multiple mathematical representations to make their learning visible to the larger group. When learning becomes visible to others, understanding will grow and ideas can be connected.

5. Better Learners of Mathematics

What the Research Says

Intelligence is perceived by many as a static condition or capacity that an individual uses to acquire, retain, and apply knowledge. But researchers such as Lev Vygotsky and Lauren Resnick argue that there are social factors that affect intelligence, and suggest that some aspects of intelligence are in fact learned. Such ideas have far-reaching implications for the design of educational programs aimed at raising the overall cognitive competence and academic achievement of educationally disadvantaged students.

Vygotsky emphasizes the role of culture in "…mediating learning—that is, in providing the tools (words, conventions, symbols, signs, etc.) through which knowledge is communicated. Knowledge is not something that individuals gain from the outside; rather it is something that they gain through their own active experiences. This means that learning and knowledge are to a large extent culturally and socially influenced" (Vygotsky, 1978).

Resnick and Sharon Nelson-Le Gall (1997) state that "interpreting intelligence as a social practice requires a critical expansion of the definition of the construct to include not just the cognitive skills and forms of knowledge that have classically been considered the essence of intelligence, but also a cluster of social performances, such as asking questions, striving to master new problems, and seeking help in problem solving…" These researchers put forth one definition of intelligence as "the ability to learn well." This definition implies that intelligence is not just knowledge or cognitive ability, but rather the process by which one acquires knowledge and cognitive ability.

Finally, research has shown that social collaboration promotes deep conceptual insights and shifts in perspective that lead to increases in student understanding and retention of concepts (Damon and Phelps, 1989; Slavin, 1996; Webb, 1989).

What Navigator Does

America's Choice believes that intelligence can be learned; as such, the Navigator program is designed with the goal of making students better learners of mathematics. Through the use of *rituals and routines* and *sharing and discussing thinking and strategies* with peers and the whole group, the Navigator classroom creates an environment that teaches students how to be better learners.

Rituals and Routines

The Navigator routines enable students to move from task to task independently, and allow the instructor to focus more on the math and less on classroom management. The rituals teach students how to interact with their classmates and the instructor around the mathematics. Consequently, the rituals and routines can help students become more productive learners who share the attitudes, habits of mind, and learning techniques of effective critical thinkers. In addition, the structure provided by the rituals and routines allows for deep and purposeful interaction with the mathematics and helps students to internalize these procedures so they become automatic.

Sharing and Discussing Thinking and Strategies

Discussing the mathematics is an important part of the Navigator program. In order to uncover and address misconceptions, students must make their thinking visible. During partner work, Checkpoint debug groups, and Probing for Understanding, students are in a safe, group setting in which they can share their thinking and strategies about the mathematics. Students benefit by gaining insight into their own thinking, as well as gaining confidence about the advantages of sharing their thoughts with their peers and the instructor.

Being able to communicate one's thoughts and ideas is important in any discipline, and students must learn how to do this in a positive and meaningful way. Being a good learner involves asking questions that will bring the appropriate feedback, as well as being able to effectively evaluate and provide feedback on the thinking of others. Navigator provides a supervised environment in which students can learn these skills. The rituals and routines, instructor guidance, and the nature of the tasks in the sessions all support the development of these collaborative learning skills.

Navigator *teaches* students to learn more effectively and become better students of mathematics. Students learn to understand each others' thinking, detect errors, talk about complex concepts, and help each other to comprehend the mathematics more fully. Navigator addresses the social as well as the intellectual dimensions of learning mathematics.

6. Instructional Support

What the Research Says

It is not uncommon for instructors who have specialized in other areas to teach mathematics, especially at the elementary level. NCES (2003) reports that "In 1999–2000, out-of-field teachers taught...23% of math students in grades 5–9 and 10% of math students in high school." However, in field tests and pilot programs, instructors have reported a need for professional development specific to mathematics learning.

Such is also the case with regard to ELL students. The demands on instructors in all disciplines are increasing as the academic arena becomes more diverse. It is not uncommon for instructors to teach English language learners without the benefit of a special credential *or* advanced knowledge of the students' primary language. Meanwhile, "the number of children ages 5 to 17 who spoke a language other than English at home more than doubled between 1979 and 2004" (NCES, 2006b). One thing is clear: instructors need instructional support in teaching English language learners.

The No Child Left Behind Act (NCLB) of 2001 guarantees ELL students an equal opportunity to achieve their learning goals. As the Virginia Department of Instruction stated in 2004, "One of the performance goals of NCLB requires students to become proficient in English while reaching high academic achievement standards in reading/language arts and mathematics." Researchers have elicited a number of important concepts in working with ELL students. Christine Denmark and Adele Neuberg (2005) point out that instructors must "continuously check for understanding by their students," and keep language "adjusted according to child's needs." Research indicates that "cooperative learning provides the conditions for nonnative English speakers who have different degrees of bilingualism to work in both languages" (Lee and Jung, 2004). And, according to Wendy Schwartz, "Asking students to devise math problems from their own experiences increases their interest, concretizes the subject, and demonstrates math's usefulness. It also promotes multiculturalism" (Schwartz, 1991).

Finally, providing meaningful assessment is a concern in any learning program. "Studies have...documented that the practice of frequent, informal monitoring (formative assessment) can produce significant learning gains, especially with low achievers" (Black and William, 1998; Black et al., 2004). Paul Black also suggests that "Assessment for learning is any assessment for which the first priority in its design and practice is to serve the purpose of promoting the pupil's learning" (Black et al., 2004). Formative assessment, in contrast to some standardized tests or district-wide exams, is meant to measure the status of a particular group of students against another, to measure a specific outcome, or to test competency, such as technical licensing exams.

What Navigator Does

The Navigator Instructor Edition is designed to provide comprehensive support to session leaders by offering detailed *instructional guidance*, *ELL support*, and a variety of both formal and informal *assessments*.

Instructional Guidance

Mathematics Navigator offers a full day of orientation and instruction. However, many instructors find that the most powerful professional support can be found in the pages of the Navigator Instructor Edition. The Instructor Edition is much more than the typical instructor guide; it is a working practicum on mathematical concepts and the facilitation of learning. The Instructor Edition offers guidance on how to lead whole-group discussions, how to monitor solo and partner work, and on many other aspects of managing a Navigator classroom. Each session includes comprehensive lists of how to prepare for the session and what materials are needed. Sessions are also preceded by a list of concrete, measurable goals, as well as the misconceptions, skills, and concepts that will be addressed. Throughout the sessions, there are teaching notes and ELL notes. Many sessions have "Errors" notes that discuss the most common errors students make in working on the types of problems in the session. The Instructor Edition provides an advanced level of support, offering explicit treatment of the concepts, tips for teaching vocabulary, suggestions for presenting tasks, and detailed procedures for dealing with misconceptions.

Perhaps the greatest instructional support is the inherent structure of Navigator sessions. The rituals, routines, and flow of the sessions do not change from day to day. Once students learn how to engage in solo and partner work and whole-group discussions, they will be able to do these tasks automatically.

ELL Support

The Instructor Edition is embedded with suggestions about how to support ELL math students. The Instructor Edition offers strategies to track English language learners' engagement in the session, encourage their participation, and check for comprehension.

The Navigator tasks are designed around research that has proven effective with ELL students. Students are often asked to modify problems or create their own problems in new contexts to share with partners. Navigator tasks use contexts and math situations that are meaningful and understandable to English language learners. Navigator students explore concepts by comparing work and exchanging ideas. Collaboration occurs with partners, in small groups, and in whole-class discussions. Mathematics Navigator is language-rich rather than language-free; math discussions occur daily and are on-going. The Instructor Edition offers effective strategies to encourage ELL students to share their knowledge.

Assessments

Multiple choice pre- and post-tests have been developed in conjunction with the Australian Council on Education Research (ACER). ACER is an international leader in the field of assessment and has been involved in both the Trends in International Math and Science Study (TIMSS) and the Programme for International Student Assessment (PISA). Formative assessments are a key component of the Navigator program. A variety of targeted monitoring techniques are used throughout the modules to inform teaching and learning. Assessments provide interim and on-going information so that instructors can adjust their instruction accordingly. Assessments, answer keys, and instructions for test administration are included in the Navigator package. Pre- and post-tests are conspicuous, but other materials and activities offer less formal assessment opportunities. These include checkpoints, peer- and self-assessments, conferences, class profiles, informal and formal observation, and student reflections on the math.

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