

Grade: 6 **Mathematics Lesson/Unit Title:**
 ▶ **enVisionmath2.0 Grade 6**
 ▶ **Topic 5 Understand and Use Ratio and Rate**

I. Alignment to the Depth of the CCSS

The lesson/unit aligns with the letter and spirit of the CCSS:

- Targets a set of grade-level CCSS mathematics standard(s) to the full depth of the standards for teaching and learning.
 - ▶ **enVisionmath2.0** Grade 6 Topic 5, Understand and Use Ratio and Rate (and Topic 6, Understand and Use Percent) focus on understanding the concepts of ratios and unit rates in the relationship between two quantities and using ratios and rates to solve real-world and mathematical problems as called for in the Grade 6 Common Core Standards. Topics 5 and 6 focus on Major Cluster 6.RP.A – Understand ratio concepts and use ratio reasoning to solve problems.
 - ▶ In Lesson 5-1, students use ratios to describe the relationship between two quantities. They use bar diagrams and double number line diagrams to model ratio relationships and solve problems. (6.RP.A.1, 6.RP.A.3)
 - ▶ In Lessons 5-2 through 5-4, students find equivalent ratios in tables, use tables to compare ratios, and represent ratios on graphs while solving real-world and mathematical problems. (6.RP.A.3a)
 - ▶ In Lesson 5-5, students use rates to describe ratios with differing units. They use rates and unit rates to solve problems. (6.RP.A.2)
 - ▶ In Lessons 5-6 and 5-7, students use ratio reasoning to compare rates and solve problems involving constant speed and unit pricing. They represent unit rate problems with equations. (6.RP.A.3b)
 - ▶ In Lessons 5-8 through 5-10, students use ratio reasoning and conversion factors to convert customary and metric units of measure. (6.RP.A.3d)
 - ▶ Continuing their understanding of ratio concepts and ratio reasoning from Topic 5, students find percents of quantities as rates per 100 in Topic 6. (6.RP.A.3c)
- Standards for Mathematical Practice that are central to the lesson are identified, handled in a grade-appropriate way, and well connected to the content being addressed.
 - ▶ **enVisionmath2.0** (Grades K–8) infuses math practices during instruction, practice, and assessment and provides opportunities to focus on specific math practices. Math practices are habits of mind, processes, and dispositions that enable a learner to understand mathematics and to use mathematics with understanding. The program identifies questions for each math practice. Math practices are infused and discussed on a daily basis starting from Day 1. In Grades 6–8, special *3-Act Mathematical Modeling* lessons are also provided as opportunities to focus on specific math practices. Math practice boxes in the *Visual Learning Bridge* include questions and comments related to the math practices and specially flagged problems in all lessons focus on specific math practices. Math practices are assessed with the content standards, and a rubric is provided for assessing students’ overall proficiency with each math practice.
 - ▶ **Math Practices and Problem Solving Handbook** This handbook at the front of the Student’s Edition provides instruction on math practices and problem solving. Refer students to appropriate parts of it anytime during a teachable moment. The Teacher’s Edition provides support for developing, connecting, and assessing each math practice. (See Student’s/Teacher’s Edition pp. F13–F27 and *Teacher’s Edition Program Overview* (Grades 6–8) pp. 44–47.)
 - ▶ **Math Practices Posters and Animations** There is a poster and animation for each math practice. They can be used anytime throughout core instruction as an aid to support discussion of a specific math practice.
 - ▶ **Core Instruction Driven by a Marriage of Content and Math Practices** Math practices are infused and explicitly highlighted in lesson instruction. First, comments related to math practices are given during problem-based learning. Then the thinking involved in math practices is modeled during direct instruction. (As an example, see Grade 6 Teacher’s Edition pp. 283A–285)

- ▶ **3-Act Mathematical Modeling Lessons** Each Grade 6–8 topic includes a lesson that focuses on mathematical modeling and specifically applies math practices. To enhance proficiency with math practices, the lesson offers opportunities to reflect on thinking related to math practices while solving a rich problem. (See Grade 6 Teacher’s Edition p. 303–304)
- ▶ **Math Practices in Problems** All math tasks evoke the selection, use, and management of multiple math practices. In the Student’s and Teacher’s Editions, red type is used to highlight problems that lend themselves to a discussion of a specific math practice.
- ▶ **Math Practices in Assessment** The program assesses math practices with content standards and provides a rubric in the Teacher’s Edition and in the *Assessment Sourcebook*. (See Grade 6 Teacher’s Edition p. F14–F15) Math Practices are also assessed in *Topic Performance Assessments*. (See Grade 6 Teacher’s Edition pp. 329C–329D)
- ▶ **Math Practices within Lessons** Math practices and content standards are connected within all lessons. The following are examples of how math practices are connected to content standards within Grade 6 Topic 5.
 - MP.1 Make sense of problems and persevere in solving them.** Students persevere as they work to understand quantities in problems involving ratios. (e.g., p. 262, Item 22)
 - MP.2 Reason abstractly and quantitatively.** Students use reasoning to analyze relationships between quantities in problems. (e.g., p. 260, Item 2)
 - MP.3 Construct viable arguments and critique the reasoning of others.** Students construct arguments to explain how a model of a ratio relationship can be used to solve problems. (e.g., p. 262, Item 21)
 - MP.4 Model with mathematics.** Students model with math when they use graphs to represent ratio relationships. (e.g., p. 278, Item 5)
 - MP.5 Use appropriate tools strategically.** Students use a multiplication chart as a tool to identify equivalent ratios. (e.g., p. 268, Item 19)
 - MP.6 Attend to precision.** Students attend to precision when they closely attend to their understanding of one quantity to verbally describe another. (e.g., p. 286, Item 2)
 - MP.7 Look for and make use of structure.** Students use structure when they identify how they might use a graph to solve problems involving ratios. (e.g., p. 278, Item 2)
 - MP.8 Look for and express regularity in repeated reasoning.** Students generalize that a unit rate is an equivalent rate with a denominator of 1. (e.g., p. 285, Example 3)

○ Presents a balance of mathematical procedures and deeper conceptual understanding inherent in the CCSS.

- ▶ **Conceptual Understanding: Understand and Attend to Two Quantities Simultaneously** Learning to pay attention to two quantities at the same time is a critical step in learning how to reason with ratios. Ratios represent a relationship, not just two numbers written together. Students develop this understanding in Grade 6 Lessons 5-1 and 5-2.
- ▶ **Conceptual Understanding: Understand Equivalent Ratios** For students to be successful in reasoning proportionally, they must understand that the ratio of two quantities remains constant even as corresponding values of the quantities change. In Grade 6 Topic 5, to assist students in developing this conceptual understanding, they repeat or iterate the basic ratio to create equivalent ratios. The relationship between the quantities is preserved when both quantities are multiplied or divided by the same nonzero number.
- ▶ **Conceptual Understanding: Special Ratios** In Grade 6 Lesson 5-5, students learn that rates are a special type of ratio in which the quantities have different units of measure, and that a unit rate is a rate in which the second quantity is one unit. The conceptual understanding that students gain about ratios applies to rates and unit rates.
- ▶ **Procedures: Use Conceptual Understanding to Find Equivalent Ratios and Unit Rates and Convert Measurement Units** Early in Grade 6 Topic 5, students find equivalent ratios by multiplying or dividing both terms by the same nonzero number. They use their understanding of equivalent ratios to compute unit rates. Later in Topic 5, using their understanding of ratios, students use dimensional analysis to convert measurement units.

II. Key Shifts in the CCSS

The lesson/unit reflects evidence of key shifts that are reflected in the CCSS:

- **Focus:** Lessons and units targeting the major work of the grade provide an especially in-depth treatment, with especially high expectations. Lessons and units targeting supporting work of the grade have visible connection to the major work of the grade and are sufficiently brief. Lessons and units do not hold students responsible for material from later grades.
 - ▶ **Focus Within a Grade: Focus on Common Core Clusters** One or more topics in **enVisionmath2.0** (Grades K–8) focus on each Common Core cluster. Grade 6 Topics 5 and 6 focus on Major Cluster 6.RP.A: Understand ratio concepts and use ratio reasoning to solve problems. Grade 6 Topic 5, *Understand and Use Ratio and Rate*, focuses on the deep understanding of recognizing, representing, and analyzing ratio relationships in verbal descriptions, tables, equations, and graphs.
 - ▶ **Focus Within a Topic: The Focus of a Topic** At the start of a topic, one or more *Essential Questions* help students focus on key ideas in the topic. The Grade 6 Topic 5 *Essential Questions* are, “What are ratios and rates? How can you use ratios and rates to describe quantities and solve problems?”
 - ▶ **Focus Within a Lesson: The Focus of a Lesson** Some of the elements of a lesson that help teachers focus students’ attention on important lesson ideas include the *Lesson Essential Question*, the *Visual Learning Bridge* and its digital counterparts the *Visual Learning Animation Plus* and other interactive examples, *Try It!*, *Convince Me!*, and the *Key Concept*.
- **Coherence:** The content develops through reasoning about the new concepts on the basis of previous understandings. Where appropriate, provides opportunities for students to connect knowledge and skills within or across clusters, domains and learning progressions.
 - ▶ *Essential Understandings* are the conceptual underpinnings of **enVisionmath2.0** (Grades K–8) and provide conceptual cohesion of the content. *Essential Understandings* connect throughout the program. An *Essential Understanding* that connects most of the work in Grade Topic 5 Major Cluster 6.RP.A is that ratios can be used to describe the relationship between two quantities where for every x units of one quantity, there are y units of another quantity.
 - ▶ **How does Grade 6 Topic 5 connect to what students learned in Grade 5?**
 - Fraction Concepts and Computations** In Grade 5, students used equivalent fractions to add and subtract fractions and mixed numbers with unlike denominators, and to multiply with fractions. They also learned to divide two whole numbers and get a quotient expressed as either a fraction or a mixed number. (5.NF.A.1, 5.NF.B.4, 5.NBT.B.6)
 - Convert Measurement Units** In Grade 5, students learned to convert measurements within a given measurement system by using multiplication or division. (5.MD.A.1)
 - Graph on the Coordinate Plane** In Grade 5, students learned to graph points in the first quadrant of the coordinate plane. (5.G.A.1)
 - ▶ **How is Grade 6 Topic 5 content connected to what students learned earlier in Grade 6?**
 - Computations with Rational Numbers** In Grade 6 Topic 1, students developed fluency in adding, subtracting, multiplying, and dividing rational numbers. (6.NS.B.2)
 - Common Factors and Multiples** In Grade 6 Topic 3, students learned how to find the greatest common factor and the least common multiple of two numbers. (6.NS.B.4)
 - Solving Equations** In Grade 6 Topic 4, students solved one-step equations with rational numbers. (6.EE.B.7)
 - ▶ **How is content connected within Grade 6 Topic 5?**
 - Ratio Reasoning** In Lessons 5-1 and 5-2, students learn to reason about ratios by using equivalent ratios, tables of equivalent ratios, bar diagrams, and double number line diagrams. They use these strategies to compare ratios in Lesson 5-3 and graph ratios in Lesson 5-4. (6.RP.A.1, 6.RP.A.3, 6.RP.A.3a)
 - Rates** In Lesson 5-5, students learn about a special type of ratio called a rate. In Lessons 5-6 and 5-7, students use their understanding of rates and their experience using tables to create equivalent rates, to compare rates, and to solve unit rate problems. In Lessons 5-8, 5-9, and 5-10, students use ratio reasoning and unit rates when converting measurements both within and between measurement systems. (6.RP.A.2, 6.RP.A.3a, 6.RP.A.3b, 6.RP.A.3d)
 - ▶ **How will Grade 6 Topic 5 connect to what students will learn later in Grade 6?**

Understand and Use Percent In Topic 6, students will understand percent as a rate in which the comparison is to 100. They will use this understanding to relate fractions, decimals, and percents and to solve problems. In Topic 8, students will use percents as they summarize data distributions. (6.RP.A.3c)

▶ **How will Grade 6 Topic 5 connect to what students will learn in Grade 7?**

Unit Rates of Fractions In Grade 7, students will compute unit rates associated with ratios of fractions. (7.RP.A.1)

Proportions In Grade 7, students will recognize and represent proportional relationships between quantities. They will also use proportional relationships to solve multistep ratio and percent problems. (7.RP.A.2)

○ **Rigor:** Requires students to engage with and demonstrate challenging mathematics with appropriate balance among the following:

- **Application:** Provides opportunities for students to independently apply mathematical concepts in real-world situations and solve challenging problems with persistence, choosing and applying an appropriate model or strategy to new situations.
- **Conceptual Understanding:** Develops students' conceptual understanding through tasks, brief problems, questions, multiple representations and opportunities for students to write and speak about their understanding.
- **Procedural Skill and Fluency:** Expects, supports and provides guidelines for procedural skill and fluency with core calculations and mathematical procedures (when called for in the standards for the grade) to be performed quickly and accurately.

▶ **Rigor in enVisionmath2.0 Grade 6 Topic 5**

Lesson 5-1, Understand Ratios – This lesson emphasizes a blend of conceptual understanding and application. Students develop a conceptual understanding of ratios as a comparison of two quantities.

Lesson 5-2, Generate Equivalent Ratios – This lesson emphasizes a blend of conceptual understanding and procedural skill. Students understand that an equivalent ratio with greater or lesser terms represents the same ratio relationship. Students learn a procedural method of generating equivalent ratios by multiplying or dividing the original terms of a ratio by the same nonzero number.

Lesson 5-3, Compare Ratios – This lesson emphasizes a blend of procedural skill and application. Students use multiplication and division to generate equivalent ratios organized in ratio tables. Students understand how to compare ratios using a common term and apply this understanding as they solve real-world and mathematical problems.

Lesson 5-4, Represent and Graph Ratios – This lesson emphasizes a blend of procedural skill and application. Students generate a table of equivalent ratios and represent the ratio relationship by plotting the corresponding pairs of values on the coordinate plane. Students use the graphs they create to solve real-world and mathematical problems.

Lesson 5-5, Understand Rates and Unit Rates – This lesson emphasizes a blend of conceptual understanding and application. Students develop an understanding of rates and unit rates and learn how to find them. Students solve real-world problems using rates and unit rates.

Lesson 5-6, Compare Unit Rates – This lesson emphasizes a blend of conceptual understanding and procedural skill. Students develop an understanding of how to use unit rates to compare two or more rates. Because the second term of all unit rates are the same, students learn that they can compare the first terms of unit rates to solve problems.

Lesson 5-7, Solve Unit Rate Problems – This lesson emphasizes a blend of procedural skill and application. Students find unit rates, including unit prices. Students apply unit rates to solve real-world problems, including those involving constant speed and unit pricing.

3-Act Mathematical Modeling: Get in Line – This mathematical modeling lesson focuses on application of both math content and math practices and processes. Students draw on their understanding of ratio and rate concepts to develop a representative model. Students apply their mathematical model to test and validate its applicability to similar problem situations.

Lesson 5-8, Ratio Reasoning: Convert Customary Units – This lesson emphasizes a blend of conceptual understanding and procedural skill. Students extend their understanding of equivalent rates when they convert measures by writing equivalent rates. Students gain an understanding of dimensional analysis and then practice determining the conversion factors needed to convert customary units of measure.

Lesson 5-9, Ratio Reasoning: Convert Metric Units – This lesson emphasizes a blend of conceptual understanding and procedural skill. Students extend their understanding of equivalent rates when they convert measures by writing equivalent rates. Students extend their work with conversion factors when they use dimensional analysis to convert metric units of measure.

Lesson 5-10, Relate Customary and Metric Units – This lesson emphasizes a blend of conceptual understanding and procedural skill. Students develop an understanding of the relationship between the customary and metric measurement systems. Students use equivalent rates and dimensional analysis to convert between systems.

▶ **Rigor embedded in the enVisionmath2.0 (Grades 6–8) Instructional Model**

enVisionmath2.0 (Grades 6–8) lessons reflect a core instructional model that supports rigor.

In Step 1 (Develop: Problem-Based Learning), the Solve & Discuss It!, Explain It!, or Explore It! problem helps students connect what they know to new ideas embedded in the problem. When students make these connections, conceptual understanding emerges. For example, in Grade 6 Lesson 5-3, p. 269, students extend their understanding of ratios to solve comparison problems.

In Step 2 (Develop: Visual Learning), teachers use the *Visual Learning Bridge*, *Visual Learning Animation Plus* and other interactive examples, and the *Key Concept* to make important lesson concepts explicit by connecting them to students' thinking and solutions from Step 1. Students demonstrate procedural skill when the procedures make sense to them. Students have an opportunity to demonstrate **procedural skill** and also **apply** their understanding through *Do You Understand?*, *Do You Know How?*, and *Practice & Problem Solving*.

Application opportunities are provided through rich, real-world problems in each lesson. For example, see Grade 6 Lesson 5-5, pp. 284–288.

In Step 3 (Assess & Differentiate), teachers can assess student understanding in the *Lesson Quiz*, and provide students with a variety of opportunities to apply and reinforce understanding in the *Reteach to Build Understanding*, *Additional Vocabulary Support*, *Build Mathematical Literacy*, *Enrichment*, *Additional Practice*, *Math Tools Activities*, and *Games*. (See Grade 6 Teacher's Edition, pp. 288A–288B)

▶ **enVisionmath2.0 (Grades 6–8) Steps to Fluency Success**

Each *Fluency Practice* activity is in one of the following formats: *Pathfinder*, *Hidden Clue*, *Crisscrossed*, or *Riddle Rearranging*. These activities can be used for practice on fluency subskills, for maintenance, or for assessment. *Math Diagnosis and Intervention System 2.0* lessons provide remediation for students who struggle with a fluency standard. (See Grade 6 Teacher's Edition p. 329 and *Teacher's Edition Program Overview* (Grades 6–8) pp. 64–65)

III. Instructional Supports

The lesson/unit is responsive to varied student learning needs:

- Includes clear and sufficient guidance to support teaching and learning of the targeted standards, including, when appropriate, the use of technology and media.

▶ **Grade 6 Topic 5 Teacher's Edition Topic Overview** (pp. 252A–252J) provides specific information on how the topic content supports Focus, Coherence, and Rigor. These pages support an in-depth understanding of the Common Core Standards for the topic.

▶ Each **Lesson Overview** (for example, Grade 6 Lesson 5-1, p. 257A) includes lesson-specific instructional support for Focus (Domain, Cluster, Content Standard, and Mathematical Practice), Coherence, and Rigor. Explicit ELL instruction to use with a specific part of each lesson is also provided. (For example, Grade 6 Teacher's Edition Lesson 5-1, p. 258)

▶ **Online lesson planning** supports lesson customization ability to upload content that is important to district initiatives.

▶ A **Professional Development Video** for each topic is provided online at PearsonRealize.com. In this *Topic Overview Video*, an author highlights and gives helpful perspectives on important mathematics concepts and skills in the topic.

▶ The online **Listen and Look For Lesson Video** provides important information about lessons in the topic.

▶ **Extensive teaching notes** support every part of every daily lesson, all lesson and topic student resources, and assessments in the topic. These include blue guiding questions to stimulate classroom discourse, incorporating math practices, preventing misconceptions, supporting coherence, error intervention, and analyzing student

work. Each lesson also includes comprehensive print and digital resources to assess and differentiate to keep all students engaged and on track. (For example, Grade 6 Teacher’s Edition Lesson 5-2, pp. 263A–268B; Lesson 5-6, pp. 289A–294B)

- Uses and encourages precise and accurate mathematics, academic language, terminology and concrete or abstract representations (e.g., pictures, symbols, expressions, equations, graphics, models) in the discipline.
 - ▶ **Vocabulary lists** for each topic are provided in the Student’s Edition (Grade 6 Topic 5, p. 252). Students encounter new vocabulary in meaningful ways as they explore concepts in the lessons. The Teacher’s Edition provides vocabulary activities at the start of topics (Grade 6 Teacher’s Edition Topic 5, p. 255). These include activities for vocabulary in *Review What You Know*. Each lesson also includes an *Additional Vocabulary Support* Blackline Master.
 - ▶ **Vocabulary review** is provided at the end of each topic, (Grade 6 Topic 5, p. 323). It reviews vocabulary used in the topic.
 - ▶ A bilingual **glossary** is provided at the back of the Student’s Edition and online to build understanding of math vocabulary.
 - ▶ An **online vocabulary game** is available in the *Game Center*.
 - ▶ Students are encouraged to represent their thinking with concrete and abstract representations throughout Grade 6 Topic 5. Students use visual models including bar diagrams for understanding ratios and rates. They use tables, double number line diagrams, write equations, and finally create graphs to show the relationships between quantities. (For an overview, see Teacher’s Edition pp. 252A–252F.)
 - ▶ The Effective Mathematics Teaching Practice “Use and Connect Mathematical Representations” as described in the **Teacher’s Edition Program Overview** (Grades 6–8, pp. 56–57) is illustrated in the Examples in each lesson in Grade 6 Topic 5 as students make connections to deepen their understanding.
 - ▶ **Bar diagrams** help students understand and represent the relationship between quantities to solve problems. (See Grade 6 Lesson 5-1 pp. 258–259 and the *Teacher’s Edition Program Overview* (Grades 6–8) pp. 48–51 for an overview.)
- Engages students in productive struggle through relevant, thought-provoking questions, problems and tasks that stimulate interest and elicit mathematical thinking.
 - ▶ **enVisionmath2.0** Grade 6 Topic 5 Opener and STEM page (pp. 253–254) introduce a *STEM Project* that students will work on for several days. Students will think like engineers as they explore the engineering design process and how gear ratios affect bicycling efficiency. Through this task, students will examine sample pairings of gears and write gear ratios to determine efficiency when designing gears for specific terrain.
 - ▶ **3-Act Mathematical Modeling Lessons** provide real-world tasks that elicit mathematical thinking and engage the Effective Mathematics Teaching Practice “Implement Tasks that Promote Reasoning and Problem Solving” as described in the Teacher’s Edition Program Overview (Grades 6–8). In Grade 6 Topic 5, students use mathematical modeling to represent a problem situation and propose a solution as they use rates to determine how long it takes a driver to get on the highway. Students test and verify the appropriateness of math models and explain why the results from mathematical models may not align exactly to problem situations as they explore possible sources of error inherent in using a math model in a real-world situation. (For example, Grade 6 Teacher’s Edition pp. 301A–304)
 - ▶ The **Step 1 Develop: Problem-Based Learning** activity for each lesson, *Solve & Discuss It!, Explain It! or Explore It!*, is designed to engage students with a problem in which new math ideas are embedded. Coherence is facilitated as students connect prior knowledge to the new math ideas. Students solve the problem in any way they choose; are given time to struggle; and as students think, conceptual understandings emerge. *Solve & Discuss It!, Explain It! or Explore It!* online utilizes the *DrawPad* to have students write and share their solutions on screen. (For example, Grade 6 Teacher’s Edition Lesson 5-2, p. 263) The questions in the Teacher’s Edition that support the *Solve & Discuss It!, Explain It!, or Explore It!* focus specifically on several of the Effective Mathematics Teaching Practices. (For example, Grade 6 Teacher’s Edition Lesson 5-8, p. 305)
 - ▶ **Visual Learning Bridge Step 2 Develop: Visual Learning** of each lesson or the online version, *Visual Learning Animation Plus* and other interactive examples, increase the cognitive level of instruction by connecting concrete and pictorial representations to abstract symbols. The questions provided in the Teacher’s Edition have

students reflect on the work that is shown, make connections among ideas, and justify the steps. The *Try It!* and *Convince Me!* features in each lesson foster communication during visual learning. (For example, Grade 6 Teacher's Edition Lesson 5-3, p. 270)

- ▶ **Practice & Problem Solving** build proficiency as students work on their own. Each Practice & Problem Solving includes a Higher Order Thinking problem. (For example, Grade 6 Teacher's Edition Lesson 5-6, p. 293–294)
- ▶ Students communicate often about their thinking and work, using reasoning during **Differentiation Lesson Activities in Part 3** of each lesson through *Enrichment*, *Digital Math Tools Activities*, and online *Math Games*. (For example, Grade 6 Teacher's Edition Lesson 5-8, p. 310B)

○ Addresses instructional expectations and is easy to understand and use.

- ▶ Clear, in-depth support for Focus, Coherence, and Rigor is found in the **Topic Overview** at the beginning of each topic in the Teacher's Edition. (For example, Grade 6 Teacher's Edition pp. 252A–252F)
- ▶ Each **Lesson Overview** provides specific information about Focus, Coherence, and Rigor for the lesson. Online lesson planning provides customization opportunities for each lesson and an online calendar. (For example, Grade 6 Lesson 5-5, p. 283A)
- ▶ The **Listen and Look For Videos** provide specific information about how the the lesson's Essential Understanding is implemented and what the teacher should look for in student work and discussions. (For example, Grade 6 Lesson 5-10, p. 317A)
- ▶ A clear, three-step lesson is provided with comprehensive teaching support for every part of the lesson – both for print and digital pathways.
- ▶ **Problem-Based Learning Solve & Discuss It!, Explain It! or Explore It!** teaching actions are provided with Before, During, and After the activity. Sample student work can be displayed online. Tips for facilitating *Problem-Based Learning* are provided and include:
 - Set Expectations – Make sure students know you expect them to do the thinking.
 - Foster communication – Have students share their thinking with a partner, small group, or the whole class.
 - Be encouraging – Show that you value students' thinking even when they struggle.
 - Use the language of the math practices during discussions.
- ▶ **Online Visual Learning Animation Plus Online and Other Interactive Examples** provide direct, stepped-out instruction. The animations, interactivities, and audio enhance learning. *Try It!* and *Convince Me!* are provided to help facilitate class discussion.
- ▶ **Guiding Questions** are provided for a question-driven classroom conversation about each part of the Visual Learning Bridge throughout each lesson.
- ▶ **Math Practices** are highlighted in red throughout the lesson.

○ Provides appropriate level and type of scaffolding, differentiation, intervention and support for a broad range of learners.

- Supports diverse cultural and linguistic backgrounds, interests and styles.
- Provides extra supports for students working below grade level.
- Provides extensions for students with high interest or working above grade level.
- ▶ **enVisionmath2.0** (Grades 6–8) meets the needs of all students and provides Response to Intervention in the following ways:
 - ▶ **Ongoing Intervention, During the Lesson**
 - Response to Intervention** during the *Visual Learning Bridge* provides additional assistance with one or more of the Examples.
 - Enrichment** during the *Visual Learning Bridge* extends one or more of the Examples to challenge students.
 - English Language Learners** during the *Visual Learning Bridge* develops and reinforces understanding of key terms and concepts.
 - Prevent Misconceptions** during the *Do You Understand?* or *Do You Know How?* sections includes a remediation strategy to address a common misconception about the lesson concept.
 - Error Intervention** during *Practice & Problem Solving* identifies a common error and provides remediation strategy.

Challenge during *Practice & Problem Solving* provides an extension of a problem's concept to enrich students' understanding.

Learning Aids Online in *MathXL for School* during the lesson includes personalized practice for *Practice & Problem Solving* for every lesson. *MathXL* is autoscored with on-screen help including *Help Me Solve This* and *View an Example*.

Higher Order Thinking problems in lesson practice challenge student thinking.

▶ **Strategic Intervention, at the End of the Lesson**

Lesson Quiz provides an assessment of the lesson's concepts and a baseline for prescribing differentiated assignments.

Reteach to Build Understanding provides guided reteaching as a follow-up to the lesson.

Additional Vocabulary Support helps students develop and reinforce understanding of key terms and concepts.

Build Mathematical Literacy provides support for struggling readers to build mathematical literacy.

Digital Math Tools Activities reinforce lesson content or previously taught content.

Online Math Games reinforce lesson content or previously taught content and include thinking games rather than just drill games. Students learn concepts through the games.

Leveled Practice & Problem Solving offers two (or three) pages of homework and practice in the Student's Edition at the end of every lesson. Items focus on skills (often with leveled practice/scaffolding for the first few exercises) and on problem solving that includes reinforcement of math practices, vocabulary, *Higher Order Thinking*, and *Common Core Assessment*.

Additional Practice provides two additional pages of Practice & Problem Solving.

Learning Aids Online in *MathXL for School* include personalized practice for every lesson. *MathXL* is autoscored with on-screen help to include *Help Me Solve This* and *View an Example*.

▶ **Intensive Intervention as Needed Anytime**

Math Diagnosis and Intervention System 2.0 Intervention Lessons contain 2 pages of guided instruction and practice.

Visual Learning Animation Plus is a step-by-step visual learning, interactive animation that is used in each lesson to connect the *Problem-Based Learning Solve & Discuss It, Explain It! or Explore It!* to the lesson concepts in a very visual way. This animation, along with the other interactive examples, can be used anytime to refresh understanding.

Online Math Games can be used anytime for more reinforcement.

Learning Aids Online MathXL for School (described earlier)

ExamView CD-ROM provides unlimited practice exercises for additional practice.

▶ **Math Diagnosis and Intervention System 2.0 (MDIS)** – Helps teachers diagnose students' needs and provides effective intervention that is on or below grade level.

Diagnosis Use the diagnostic tests in the system. Also, use the item analysis charts given with program assessments at the start of a grade or topic, or at the end of a topic, group of topics, or the year.

Intervention Lessons These two-page lessons include guided instruction followed by practice. You can assign lessons that are below grade level if needed.

Teacher Support Teacher Notes provide the support needed to conduct a short lesson. The lesson focuses on vocabulary, concept development, and practice. The Teacher's Guide contains individual and class record forms and correlations to Student's Edition lessons.

▶ **PearsonRealize.com Auto Assigned Differentiation**

Differentiation Before a Topic is based on the result of *Topic Readiness Tests*. Students may be assigned intervention lessons, which include lesson Journal PDFs and MathXL Practice.

Differentiation After a Lesson is based on results of the *Lesson Quiz*. Students are given the Intervention or Advanced assignment based on student performance. The Intervention assignment includes a Reteach to Build Understanding master while the Advanced assignment includes an Enrichment master.

Differentiation After a Group of Lessons is based on the results of the *Mid-Topic Assessment*. Students may be assigned to review specific lesson concepts digitally or to view the Virtual Nerd instructional tutorials.

Differentiation After a Topic or Group of Topics is based on results of online *Topic Assessments* and online *Cumulative/Benchmark Assessments*. Students are assigned remediation which includes digital review of specific

lesson concepts and Intervention Lessons in the *Math Diagnosis and Intervention System 2.0*. At any time, a teacher may assign students digital math games or Digital Math Tools Activities as needed for differentiation.

Assignment Reports show the status of assigned resources.

Usage Data lets teachers know how much time students are spending in the online resources.

- ▶ **Today's Challenge Online** shows 5 problems using the same data on 5 different days. Problems apply prior knowledge and reinforce the kind of thinking students need for success on high-stakes tests. Problems increase in difficulty within a set. *Today's Challenge Teacher's Guide* includes teaching actions organized under Before, During, and After in addition to Vocabulary Review, ELL Support, and Extension.
- ▶ **English Language Learners** daily ELL instruction support is provided in the Teacher's Edition for every lesson. This support is used with a specified part of the lesson such as *Solve & Discuss It!*, *Explain It!*, or *Explore It!*, *Visual Learning Bridge*, *Try It!*, or *Convince Me!*. Leveled instruction includes suggestions for students at Beginning, Intermediate, and Advanced levels of English Language Proficiency. ELL Toolkit provides additional support for English Language Learners.
- ▶ **Visual Learning Animation Plus and Other Interactive Examples Online** provide motion and sound to help lower language barriers to learning. Questions that are read aloud also appear on screen to help English Language Learners connect oral and written language.
- ▶ **Visual Learning Bridge** often has visual models to help give meaning to math language. Instruction is stepped out to visually organize important ideas. Teachers may want to use the Student's Edition eText to display the *Visual Learning Bridge* after using the *Visual Learning Animation Plus* and other interactive examples.
- ▶ **Online Glossary** is always available to students and teachers while using digital resources. The glossary is in English and Spanish to help students connect Spanish math terms they may know to English equivalents.
- ▶ **Pictures with a purpose** that appear in lesson practice provide comprehensible input and help communicate information related to math concepts or to real-world problems.

A unit or longer lesson should:

- Recommend and facilitate a mix of instructional approaches for a variety of learners such as using multiple representations (e.g., including models, using a range of questions, checking for understanding, flexible grouping, pair-share).
 - ▶ Students frequently use models to represent their thinking and solutions in the **Develop: Problem-Based Learning** Part 1 of each lesson, **Solve & Discuss It!, Explain It!, or Explore It!**. Teachers are provided with guiding questions to facilitate this discussion and **Before**, **During**, and **After** teaching actions model classroom discourse.
 - Before** uses Teaching Actions #1 (Implement Tasks that Promote Reasoning and Problem Solving) and #2 (Build Understanding) to start understanding. This is whole-class discussion.
 - During** (#3 Support Productive Struggle in Learning Mathematics) is used when students are stumped and are working together in pairs or small groups as the teacher facilitates.
 - After** represents another whole-class discussion. Use teaching action #4 (Facilitate Meaningful Mathematical Discourse) and #5 (Transition to Visual Learning) to discuss students' thinking and work, and to make math ideas explicit. Use #6 (Extension for Early Finishers) as needed.
 - ▶ Blue guiding questions are provided throughout each lesson to support classroom discourse.
 - ▶ *Try It!* and *Convince Me!* checks for understanding right after the first example and connects to the essential understanding of the lesson and to math practices.
 - ▶ A variety of activities are provided for differentiation where students may be working with the teacher in small groups or in pairs on a digital device or in print. Different learning modalities offer greater access for all students.
- Gradually remove supports, requiring students to demonstrate their mathematical understanding independently.
 - ▶ The **enVisionmath2.0** (Grades 6–8) instructional lesson design (print and online) represents a gradual release model.
 - Step 1: Develop: Problem-Based Learning, Solve & Discuss It!, Explain It! or Explore It!*, introduces the lesson by engaging students with a problem in which new math ideas are embedded. Students work in small groups to solve the problem with teacher-facilitated discussion.

- *Step 2: Develop: Visual Learning* is provided whole class through direct instruction from the *Visual Learning Bridge* and/or *Visual Learning Animation Plus* and other interactive examples online where learning is stepped out. *Do You Understand? Do You Know How?* includes teacher-supported concepts and skills practice. *Practice & Problem Solving* builds proficiency as students work on their own.

-*Step 3: Assess & Differentiate* provides additional independent work through *Leveled Additional Practice* – two additional pages of homework and practice.

- ▶ *Practice & Problem Solving* items focus on skills (often with leveled practice/scaffolding for the first few exercises) and on problem solving that includes reinforcement of math practices, vocabulary, *Higher Order Thinking*, and *Common Core Assessment*.
- ▶ *Additional Practice* items focus on skills (often with scaffolding) and on problem solving that includes reinforcement of math practices, vocabulary, *Higher Order Thinking*, and *Common Core Assessment*.
- ▶ *Reteach to Build Understanding* provides scaffolded, guided reteaching that prepares students for the homework.

○ Demonstrate an effective sequence and a progression of learning where the concepts or skills advance and deepen over time.

- ▶ One or more topics in **enVisionmath2.0** (Grades K–8) focus on each Common Core cluster. Grade 6 Topics 5 and 6 focus on Major Cluster 6.RP.A: Understand ratio concepts and use ratio reasoning to solve problems. Grade 6 Topic 5, Understand and Use Ratio and Rate, focuses on the deep understanding of ratios and rates and the ways they can be used to solve problems. Topic 6, Understand and Use Percent, then builds on and extends that understanding to percents.
- ▶ At the start of a topic, one or more *Essential Questions* help students focus on key ideas in the topic. Grade 6 Topic 5 *Essential Questions* are, “What are ratios and rates? How can you use ratios and rates to describe quantities and solve problems?” Topic 6 *Essential Questions* are, “What is the meaning of percent? How can percent be estimated and found?”
- ▶ An *Essential Understanding* is stated in the Teacher’s Edition for each lesson. The Essential Understandings build from each other and show the progression of concepts within a topic.
- ▶ Some of the elements of a lesson that help teachers focus students’ attention on important lesson ideas include the *Lesson Essential Question*, the *Visual Learning Bridge* and its digital counterpart, the *Visual Learning Animation Plus* and other interactive examples, the *Key Concept*, as well as the *Try It!* or *Convince Me!*.

○ Expect, support and provide guidelines for procedural skill and fluency with core calculations and mathematical procedures (when called for in the standards for the grade) to be performed quickly and accurately.

- ▶ **enVisionmath2.0 (Grades K–8) provides steps to fluency success.** Students achieve fluency when they demonstrate skill in carrying out procedures flexibly, accurately, efficiently, and appropriately. Follow the *Steps to Fluency Success* to help all students achieve fluency standard expectations for each grade.

Step 1: Fluency Development with Understanding For each fluency standard, there is a topic that develops fluency with understanding. That topic is the culmination of foundations for fluency taught previously.

Step 2: Ongoing Assessment of Fluency Subskills After Step 1, assess fluency subskills using one of the following:

Fluency Practice/Assessment Worksheets (Grades K–5) in Teacher’s Resource Masters

Fluency Practice/Assessment Worksheets (Grades K–5) generated by ExamView® CD-ROM

Practice Buddy Online (Grades 3–5) or **MathXL for School** (Grades 6–8) at *PearsonRealize.com* After Step 2, for students who are fluent, go to Step 5.

Step 3: Fluency Intervention For students who struggle with a fluency standard, use one of the following:

MDIS (Math Diagnosis and Intervention System) Lessons for remediation of subskills. You can also give an *MDIS Diagnostic Test* to assess a need for *MDIS lessons* on prerequisite skills.

Practice Buddy Online (Grades 3–5) or **MathXL for School** (Grades 6–8) with intervention also available via the learning aids “*Help Me Solve This*” and “*View an Example.*”

Step 4: Practice on Fluency Subskills After Step 3, give students practice using one of the following. Then reassess (Step 2).

Fluency Practice/Assessment Worksheets (Grades K–5) in Teacher’s Resource Masters

Fluency Practice/Assessment Worksheets (Grades K–5) generated by ExamView® CD-ROM
Practice Buddy Online (Grades 3–5) or **MathXL for School** (Grades 6–8)

Step 5: Fluency Maintenance For fluency maintenance, use some of the following:

Fluency Practice Activities in the Student’s Edition. Students can collaborate, allowing them to help each other as needed.

Fluency Practice/Assessment Worksheets (Grades K–5) in Teacher’s Resource Masters

Fluency Practice/Assessment Worksheets (Grades K–5) generated by ExamView® CD-ROM

Practice Buddy Online (Grades 3–5) or **MathXL for School** (Grades 6–8)

Game Center Online at *PearsonRealize.com*

Step 6: Summative Fluency Assessment For a summative fluency assessment of all students, use one of the following:

Fluency Practice/Assessment Worksheets (Grades K–5) in Teacher’s Resource Masters

Fluency Practice/Assessment Worksheets (Grades K–5) generated by ExamView® CD-ROM

Practice Buddy Online (Grades 3–5) or **MathXL for School** (Grades 6–8)

IV. Assessment

The lesson/unit regularly assesses whether students are mastering standards-based content and skills:

- Is designed to elicit direct, observable evidence of the degree to which a student can independently demonstrate the targeted CCSS.
- Assesses student proficiency using methods that are accessible and unbiased, including the use of grade-level language in student prompts.
- Includes aligned rubrics, answer keys and scoring guidelines that provide sufficient guidance for interpreting student performance.

▶ **enVisionmath2.0** (Grades K–8) provides opportunities for students to independently demonstrate their understanding.

Online assessments can be customized as needed.

Math Practices Proficiency Rubric is provided and can be used at any time.

Scoring rubrics and sample student work are included for some assessments in the *Assessment Sourcebook*.

▶ The formats of the assessment items, which prepare students for high-stakes Common Core tests, include the following:

Selected response, e.g., single response, multiple response

Constructed response, e.g., short or extended responses, sometimes using an on-screen symbols palette

Technology-enhanced items, e.g., drag and drop, drop-down menus, graphing, on-screen tools

Performance tasks, hand scored or machine scored

▶ A variety of auto-generated assessment reports are available for online assessments.

Individual and class views of progress are provided in an easy-to-view format.

Common Core Standards mastery reports show individual students’ mastery or class-wide mastery for each standard.

▶ **Assessment data** can be used to organize students into groups for purposes of making instructional decisions and assigning differentiation resources.

▶ Every lesson includes **Common Core practice items** in formats that help prepare students for Common Core high-stakes tests. These types of items are also included in Readiness Assessments, Mid-topic Checkpoints and Performance Tasks, Lesson Quizzes, End of Topic Assessments and Performance Tasks.

A unit or longer lesson should:

- Use varied modes of curriculum-embedded assessments that may include pre-, formative, summative and self-assessment measures.

▶ **enVisionmath2.0** (Grades K–8) provides students with multiple ways to show what they have learned.

▶ **DIAGNOSTIC ASSESSMENT**

At the start of the YEAR

Beginning-of-Year Assessment Online in TestNav™
Beginning-of-Year Assessment Masters in Assessment Sourcebook
Diagnostic Test Masters in Math Diagnosis and Intervention System 2.0

At the start of a TOPIC

Topic Readiness Assessment Online in TestNav™
Topic Readiness Assessment Masters in Assessment Sourcebook
Diagnostic Test Masters in Math Diagnosis and Intervention System 2.0
Review What You Know in Student's Edition or eText

▶ **FORMATIVE ASSESSMENT**

During a LESSON

Questions in the *Visual Learning Animation Plus* and *Other Interactive Examples*
Questions to use with the *Visual Learning Bridge* in Teacher's Edition
Try It! and *Convince Me!* in Student's Edition or eText
Do You Understand? Do You Know How? in Student's Edition or eText

At the end of a LESSON

Lesson Quiz Online in TestNav™
Lesson Quiz Masters in Assessment Sourcebook

▶ **SUMMATIVE ASSESSMENT**

In the middle of a TOPIC

Mid-Topic Checkpoint in Student's Edition or eText
Mid-Topic Assessment Online in TestNav™
Mid-Topic Assessment Masters in Assessment Sourcebook
Self-Assessment Tool in Mid-Topic Assessment
Mid-Topic Performance Task in Student's Edition or eText

At the end of a TOPIC

Topic Assessment Masters in Assessment Sourcebook
Topic Assessment Online in TestNav™
Topic Assessment by ExamView® CD-ROM
Topic Performance Assessment Masters in Assessment Sourcebook
Fluency Assessment by Practice Buddy Online (Grades K–5)
Fluency Practice/Assessment Masters in Teacher's Resource Masters (Grades K–5)
Basic-Facts Timed Test Masters (Grades 1–5) in *Assessment Sourcebook*

After a group of TOPICS

Cumulative/Benchmark Assessment Online in TestNav™
Cumulative/Benchmark Assessment Masters in Assessment Sourcebook
Practice Performance Tasks Assessment Online in TestNav™
Practice Performance Tasks Assessment Masters in Assessment Sourcebook

At the end of the YEAR

End-of-Year Assessment Online in TestNav™
End-of-Year Assessment Masters in Assessment Sourcebook Masters
Next Generation Assessment Practice Test Online in TestNav™
Next Generation Assessment Practice Test Assessment Masters in Assessment Sourcebook

▶ **SELF ASSESSMENT**

Self-Assessment Tool in Teaching Tools section of the *Teacher's Resource Masters* (Grades K–8)
Self-Assessment Tool in Mid-Topic Assessment in *Assessment Sourcebook*

The EQulP rubric is derived from the Tri-State Rubric and the collaborative development process led by Massachusetts, New York, and Rhode Island and facilitated by Achieve.

This version of the EQulP rubric is current as of 06-15-13.

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