

Grade: 7 Mathematics Lesson/Unit Title:

- ▶ **enVisionmath2.0 Grade 7**
- ▶ **Topic 2 Analyze and Use Proportional Relationships**

I. Alignment to the Depth of the CCSS

The lesson/unit aligns with the letter and spirit of the CCSS:

- Targets a set of grade-level CCSS mathematics standard(s) to the full depth of the standards for teaching and learning.
 - ▶ **enVisionmath2.0** Grade 7 Topic 2, Analyze and Use Proportional Relationships (and Topic 3, Analyze and Solve Percent Problems) focus on recognizing and representing proportional relationships in verbal descriptions, tables, equations, and graphs and using them to solve real-world and mathematical problems as called for in the Grade 7 Common Core Standards. Topics 2 and 3 focus on Major Cluster 7.RP.A — Analyze proportional relationships and use them to solve real-world and mathematical problems.
 - ▶ In Lesson 2-1, students find, use, and compare unit rates associated with ratios of whole numbers and decimals in multistep problems with quantities measured in like or different units. In Lesson 2-2, students compute unit rates for ratios with rational terms by applying the standard algorithm for division of fractions. (7.RP.A.1, 7.RP.A.3)
 - ▶ In Lesson 2-3, students use descriptions, tables, and diagrams to determine whether two quantities are in a proportional relationship and use proportional relationships to solve problems. (7.RP.A.2a) In Lesson 2-5, students recognize proportional relationships in graphs (7.RP.A.2a) and explain how the quantities in the graph are related by interpreting the meaning of the points (x, y) , $(0, 0)$, and $(1, y)$. (7.RP.A.2d)
 - ▶ In Lesson 2-4, students represent proportional relationships and identify the constant of proportionality in tables, diagrams, and descriptions and with the equation $y = kx$ where k is the constant of proportionality. (7.RP.A.2b, 7.RP.A.2c)
 - ▶ Throughout Topic 2, students recognize when to use and apply proportional reasoning to solve problems. (7.RP.A.2, 7.RP.A.3)
- Standards for Mathematical Practice that are central to the lesson are identified, handled in a grade-appropriate way, and well connected to the content being addressed.
 - ▶ **enVisionmath2.0** (Grades K–8) infuses math practices during instruction, practice, and assessment and provides opportunities to focus on specific math practices. Math practices are habits of mind, processes, and dispositions that enable a learner to understand mathematics and to use mathematics with understanding. The program identifies questions for each math practice. Math practices are infused and discussed on a daily basis starting from Day 1. In Grades 6–8, special *3-Act Mathematical Modeling* lessons are also provided as opportunities to focus on specific math practices. Math practice boxes in the *Visual Learning Bridge* include questions and comments related to the math practices and specially flagged problems in all lessons focus on specific math practices. Math practices are assessed with the content standards, and a rubric is provided for assessing students' overall proficiency with each math practice.
 - ▶ **Math Practices and Problem Solving Handbook** This handbook at the front of the Student's Edition provides instruction on math practices and problem solving. Refer students to appropriate parts of it anytime during a teachable moment. The Teacher's Edition provides support for developing, connecting, and assessing each math practice. (See Student's/Teacher's Edition pp. F13–F27 and *Teacher's Edition Program Overview* (Grades 6–8) pp. 44–47.)
 - ▶ **Math Practices Posters and Animations** There is a poster and animation for each math practice. They can be used anytime throughout core instruction as an aid to support discussion of a specific math practice.
 - ▶ **Core Instruction Driven by a Marriage of Content and Math Practices** Math practices are infused and explicitly highlighted in lesson instruction. First, comments related to math practices are given during problem-based

learning. Then the thinking involved in math practices is modeled during direct instruction. (As an example, see Grade 7 Teacher’s Edition pp. 91A–93)

- ▶ **3-Act Mathematical Modeling Lessons** Each Grade 6–8 topic includes a lesson that focuses on mathematical modeling and specifically applies math practices. To enhance proficiency with math practices, the lesson offers opportunities to reflect on thinking related to math practices while solving a rich problem. (See Grade 7 Teacher’s Edition p. 113–114)
- ▶ **Math Practices in Problems** All math tasks evoke the selection, use, and management of multiple math practices. In the Student’s and Teacher’s Editions, red type is used to highlight problems that lend themselves to a discussion of a specific math practice.
- ▶ **Math Practices in Assessment** The program assesses math practices with content standards and provides a rubric in the Teacher’s Edition and in the *Assessment Sourcebook*. (See Grade 7 Teacher’s Edition p. F14–F15) Math Practices are also assessed in *Topic Performance Assessments*. (See Grade 7 Teacher’s Edition pp. 131C–131D)
- ▶ **Math Practices within Lessons** Math practices and content standards are connected within all lessons. The following are examples of how math practices are connected to content standards within Grade 7 Topic 2.
 - MP.1 Make sense of problems and persevere in solving them.** Students persevere as they try to understand problems involving equivalent ratios. (e.g., p. 89, Item 12)
 - MP.2 Reason abstractly and quantitatively.** Students use reasoning to analyze relationships between quantities in problems. (e.g., p. 100, Item 3)
 - MP.3 Construct viable arguments and critique the reasoning of others.** Students construct arguments to justify that the relationship between two quantities is proportional. (e.g., p. 101, Item 8)
 - MP.4 Model with mathematics.** Students model with math when they use equations to represent problems involving proportional relationships. (e.g., p. 107, Item 10)
 - MP.5 Use appropriate tools strategically.** Students identify tools to represent quantities in proportional relationships. (e.g., p. 124, Item 2)
 - MP.6 Attend to precision.** Students attend to precision when they closely attend to quantities in a problem and choose the correct units in their answer. (e.g., p. 89, Item 11)
 - MP.7 Look for and make use of structure.** Students use structure when they identify what a point represents in a graph of a line representing a proportional relationship. (e.g., p. 119, Item 11a)
 - MP.8 Look for and express regularity in repeated reasoning.** Students generalize about using equations to represent unknowns in proportional relationships. (e.g., p. 106, Item 2)

○ Presents a balance of mathematical procedures and deeper conceptual understanding inherent in the CCSS.

- ▶ **Conceptual Understanding: Understand Proportional Relationships** In Grade 7 Topic 2, students build on their understanding of equivalent ratios and rates and determine whether two quantities are in a proportional relationship. Students learn that the relationship between two quantities is preserved when both quantities are multiplied by the same number. This approach develops the understanding that the constant of proportionality is the constant multiple that relates the proportional quantities.
- ▶ **Conceptual Understanding: Understand Proportional Relationships Can Be Represented by Equations and Graphs** In Grade 7 Lesson 2-4, students extend their understanding of proportional relationships to develop the equation $y = kx$ to represent the proportional relationship between the quantities x and y , where k is the constant of proportionality. In Grade 7 Lesson 2-5, students use their understanding of proportional relationships to explain what a point (x, y) means with special attention to the points $(0, 0)$ and $(1, r)$, where r is the unit rate.
- ▶ **Procedures: Compute Unit Rates Associated with Ratios of Fractions** Early in Grade 7 Topic 2, students find equivalent ratios by multiplying or dividing both terms by the same nonzero number. They use their understanding of equivalent ratios to compute unit rates with complex fractions.

II. Key Shifts in the CCSS

The lesson/unit reflects evidence of key shifts that are reflected in the CCSS:

- **Focus:** Lessons and units targeting the major work of the grade provide an especially in-depth treatment, with especially high expectations. Lessons and units targeting supporting work of the grade have visible connection to the major work of the grade and are sufficiently brief. Lessons and units do not hold students responsible for material from later grades.
 - ▶ **Focus Within a Grade: Focus on Common Core Clusters** One or more topics in **enVisionmath2.0** (Grades K–8) focus on each Common Core cluster. Grade 7 Topics 2 and 3 focus on Major Cluster 7.RP.A: Analyze proportional relationships and use them to solve real-world and mathematical problems. Grade 7 Topic 2, *Analyze and Use Proportional Relationships*, focuses on the deep understanding of recognizing, representing, and analyzing proportional relationships in verbal descriptions, tables, equations, and graphs.
 - ▶ **Focus Within a Topic: The Focus of a Topic** At the start of a topic, one or more *Essential Questions* help students focus on key ideas in the topic. The Grade 7 Topic 2 *Essential Question* is, “How can you recognize and represent proportional relationships and use them to solve problems?”
 - ▶ **Focus Within a Lesson: The Focus of a Lesson** Some of the elements of a lesson that help teachers focus students’ attention on important lesson ideas include the *Lesson Essential Question*, the *Visual Learning Bridge* and its digital counterparts the *Visual Learning Animation Plus* and other interactive examples, *Try It!*, *Convince Me!*, and the *Key Concept*.

- **Coherence:** The content develops through reasoning about the new concepts on the basis of previous understandings. Where appropriate, provides opportunities for students to connect knowledge and skills within or across clusters, domains and learning progressions.
 - ▶ *Essential Understandings* are the conceptual underpinnings of **enVisionmath2.0** (Grades K–8) and provide conceptual cohesion of the content. *Essential Understandings* connect throughout the program. An *Essential Understanding* that connects most of the work in Grade 7 Topic 2 Major Cluster 7.RP.A is that by recognizing proportional quantities, students can use what they know about proportional relationships to solve problems.
 - ▶ **How does Grade 7 Topic 2 connect to what students learned in Grade 6?**
 - Ratio Reasoning** In Grade 6, students reasoned about ratios by using equivalent ratios, tables of equivalent ratios, bar diagrams, and double number line diagrams. In Grade 6 Topic 6, students used what they learned about ratios to work with a special type of ratio called a percent. (6.RP.A.3, 6.RP.A.3c)
 - Rates** In Grade 6 Topic 5, students learned about a special type of ratio called a rate. (6.RP.A.2)
 - ▶ **How is Grade 7 Topic 2 content connected to what students learned earlier in Grade 7?**
 - Complex Fractions** In Grade 7 Topic 1, students learned to divide rational numbers, including simplifying complex fractions, which prepared students for computing unit rates associated with ratios of fractions in Topic 2. (7.NS.A)
 - ▶ **How is content connected within Grade 7 Topic 2?**
 - Ratios and Rates** In Lesson 2-1, students make connections between ratios, rates, and unit rates. In Lesson 2-2, students determine unit rates with ratios of fractions. (7.RP.A.1)
 - Proportional Relationships** In Lesson 2-3, students use equivalent ratios to determine whether relationships are proportional. They write and solve proportions to answer questions about situations involving proportional relationships. In Lesson 2-4, they use the constant of proportionality to write equations that describe proportional relationships. In Lesson 2-5, students graph proportional relationships. In Lesson 2-6, they use proportional reasoning in problem-solving contexts. (7.RP.A.2, 7.RP.A.3)
 - ▶ **How will Grade 7 Topic 2 connect to what students will learn later in Grade 7?**
 - Percents** In Topic 3, students will connect percent to proportional reasoning. They will use what they know about writing equations of proportional relationships to represent and use the percent equation and solve problems that include percent change, percent error, markup, markdown, and simple interest. (7.RP.A.3)
 - ▶ **How will Grade 7 Topic 2 connect to what students will learn in Grade 8?**
 - Proportional Relationships** In Grade 8, students will understand the connections among proportional relationships, lines, and linear equations. Students will also graph proportional relationships and compare proportional relationships that are represented in different ways. (8.EE.B.5)
 - Similarity** In Grade 8, students will use proportional relationships with similar triangles to investigate slope, and they will derive the equations $y = mx$ and $y = mx + b$ for lines. (8.EE.B.6)

- **Rigor:** Requires students to engage with and demonstrate challenging mathematics with appropriate balance among the following:
 - **Application:** Provides opportunities for students to independently apply mathematical concepts in real-world situations and solve challenging problems with persistence, choosing and applying an appropriate model or strategy to new situations.
 - **Conceptual Understanding:** Develops students’ conceptual understanding through tasks, brief problems, questions, multiple representations and opportunities for students to write and speak about their understanding.
 - **Procedural Skill and Fluency:** Expects, supports and provides guidelines for procedural skill and fluency with core calculations and mathematical procedures (when called for in the standards for the grade) to be performed quickly and accurately.

▶ **Rigor in enVisionmath2.0 Grade 7 Topic 2**

Lesson 2-1, Connect Ratios, Rates, and Unit Rates – This lesson emphasizes a blend of conceptual understanding and application. Students understand and reason about ratios and rates, and make comparisons with unit rates. Students apply what they know about equivalent ratios and unit rates to solve multi-step problems in real-world situations.

Lesson 2-2, Determine Unit Rates with Ratios of Fractions – This lesson emphasizes a blend of conceptual understanding and procedural skill. Students relate the use of a table of equivalent ratios to the procedure of dividing by the second term of a ratio, or multiplying by its reciprocal, to find the associated unit rate. Students extend their understanding and skills with unit rates and fraction operations as they solve problems involving ratios of fractions.

Lesson 2-3, Understand Proportional Relationships: Equivalent Ratios – This lesson emphasizes a blend of conceptual understanding and procedural skill. Students extend their prior understanding of a unit rate and explore its relationship to equivalent ratios. Students look for equivalent ratios to determine if a relationship is proportional.

Lesson 2-4, Describe Proportional Relationships: Constant of Proportionality – This lesson emphasizes a blend of conceptual understanding and procedural skill. Students learn how to represent a proportional relationship with an equation using the constant of proportionality. Students use equations to solve problems involving proportional relationships.

Lesson 2-5, Graph Proportional Relationships – This lesson emphasizes a blend of conceptual understanding and application. Students understand the characteristics of a graph of a proportional relationship. Students interpret points on graphs of proportional relationships in the context of real-world situations.

3-Act Mathematical Modeling: Mixin’ It Up – This mathematical modeling lesson focuses on application of both math content and math practices and processes. Students draw on their understanding of ratio and proportionality concepts to develop a representative model. Students apply their mathematical model to test and validate its applicability to similar problem situations.

Lesson 2-6, Apply Proportional Reasoning to Solve Problems – This lesson emphasizes a blend of conceptual understanding and procedural skill. Students use reasoning skills to determine whether a relationship is proportional. Students apply what they know about additive and multiplicative relationships to solve problems.

▶ **Rigor embedded in the enVisionmath2.0 (Grades 6–8) Instructional Model**

enVisionmath2.0 (Grades 6–8) lessons reflect a core instructional model that supports rigor.

In Step 1 (Develop: Problem-Based Learning), the Solve & Discuss It!, Explain It!, or Explore It! problem helps students connect what they know to new ideas embedded in the problem. When students make these connections, conceptual understanding emerges. For example, in Grade 7 Lesson 2-2, p. 91, students extend their understanding of ratios, rates, and unit rates to solve a comparison problem with differing units.

In Step 2 (Develop: Visual Learning), teachers use the *Visual Learning Bridge*, *Visual Learning Animation Plus* and other interactive examples, and the *Key Concept* to make important lesson concepts explicit by connecting them to students’ thinking and solutions from Step 1. Students demonstrate procedural skill when the procedures make sense to them. Students have an opportunity to demonstrate **procedural skill** and also **apply** their understanding through *Do You Understand?*, *Do You Know How?*, and *Practice & Problem Solving*.

Application opportunities are provided through rich, real-world problems in each lesson. For example, see Grade 7 Lesson 2-3, pp. 98–102.

In Step 3 (Assess & Differentiate), teachers can assess student understanding in the *Lesson Quiz*, and provide students with a variety of opportunities to apply and reinforce understanding in the *Reteach to Build Understanding*, *Additional Vocabulary Support*, *Build Mathematical Literacy*, *Enrichment*, *Additional Practice*, *Digital Math Tools Activities*, and *Games*. (See Grade 7 Teacher’s Edition, pp. 102A–102B)

▶ **enVisionmath2.0 (Grades 6–8) Steps to Fluency Success**

Each *Fluency Practice* activity is in one of the following formats: *Pathfinder*, *Hidden Clue*, *Crisscrossed*, or *Riddle Rearranging*. These activities can be used for practice on fluency subskills, for maintenance, or for assessment. *Math Diagnosis and Intervention System 2.0* lessons provide remediation for students who struggle with a fluency standard. (See Grade 7 Teacher’s Edition p. 131 and *Teacher’s Edition Program Overview (Grades 6–8)* pp. 64–65)

III. Instructional Supports

The lesson/unit is responsive to varied student learning needs:

- Includes clear and sufficient guidance to support teaching and learning of the targeted standards, including, when appropriate, the use of technology and media.
 - ▶ **Grade 7 Topic 2 Teacher’s Edition Topic Overview** (pp. 80A–80H) provides specific information on how the topic content supports Focus, Coherence, and Rigor. These pages support an in-depth understanding of the Common Core Standards for the topic.
 - ▶ Each **Lesson Overview** (for example, Grade 7 Lesson 2-1, p. 85A) includes lesson-specific instructional support for Focus (Domain, Cluster, Content Standard, and Mathematical Practice), Coherence, and Rigor. Explicit ELL instruction to use with a specific part of each lesson is also provided. (For example, Grade 7 Teacher’s Edition Lesson 2-1, p. 86)
 - ▶ **Online lesson planning** supports lesson customization ability to upload content that is important to district initiatives.
 - ▶ A **Professional Development Video** for each topic is provided online at PearsonRealize.com. In this *Topic Overview Video*, an author highlights and gives helpful perspectives on important mathematics concepts and skills in the topic.
 - ▶ The online **Listen and Look For Lesson Videos** provide important information about lessons in the topic.
 - ▶ **Extensive teaching notes** support every part of every daily lesson, all lesson and topic student resources, and assessments in the topic. These include blue guiding questions to stimulate classroom discourse, incorporating math practices, preventing misconceptions, supporting coherence, error intervention, and analyzing student work. Each lesson also includes comprehensive print and digital resources to assess and differentiate to keep all students engaged and on track. (For example, Grade 7 Teacher’s Edition Lesson 2-3, pp. 97A–102B; Lesson 2-6 pp. 121A–126B)
 - ▶ The **Teacher’s Edition Program Overview** (Grades 6–8, pp. 56–57) describes the Effective Mathematics Teaching Practices that guided the development of **enVisionmath2.0**. Throughout each lesson are questions and strategies that focus on each of the eight teaching practices to support teaching and learning of the standards.
- Uses and encourages precise and accurate mathematics, academic language, terminology and concrete or abstract representations (e.g., pictures, symbols, expressions, equations, graphics, models) in the discipline.
 - ▶ **Vocabulary lists** for each topic are provided in the Student’s Edition (Grade 7 Topic 2, p. 80). Students encounter new vocabulary in meaningful ways as they explore concepts in the lessons. The Teacher’s Edition provides vocabulary activities at the start of topics (Grade 7 Teacher’s Edition Topic 2, pp. 83–84). These include activities for vocabulary in *Review What You Know* and *Build Vocabulary*. Each lesson also includes an *Additional Vocabulary Support* Blackline Master.
 - ▶ **Vocabulary review** is provided at the end of each topic, (Grade 7 Topic 2, p. 127). It reviews vocabulary used in the topic.
 - ▶ A bilingual **glossary** is provided at the back of the Student’s Edition and online to build understanding of math vocabulary.
 - ▶ An **online vocabulary game** is available in the *Game Center*.

- ▶ Students are encouraged to represent their thinking with concrete and abstract representations throughout Grade 7 Topic 2. Students use visual models such as drawings or illustrations for understanding ratios, rates, and proportional relationships. They use tables, write expressions and equations, and finally create graphs to show the relationships between quantities. (For an overview, see Teacher’s Edition pp. 80A–80F.)
- ▶ The Effective Mathematics Teaching Practice “Use and Connect Mathematical Representations” as described in the **Teacher’s Edition Program Overview** (Grades 6–8, pp. 56–57) is illustrated in the Examples in each lesson in Grade 7 Topic 2 as students make connections to deepen their understanding of new concepts.
- ▶ **Bar diagrams** help students understand and represent the relationship between quantities to solve problems. (See Grade 7 Lesson 2-6 p. 122 and the *Teacher’s Edition Program Overview* (Grades 6–8) pp. 48–51 for an overview.)

○ Engages students in productive struggle through relevant, thought-provoking questions, problems and tasks that stimulate interest and elicit mathematical thinking.

- ▶ **enVisionmath2.0** Grade 7 Topic 2 Opener and STEM page (pp. 81–82) introduce a *STEM Project* that students will work on for several days. They will do research on access to fresh, clean water, explore methods to gain access to clean water, and brainstorm a solution for how much freshwater can be produced with a filtration system. Through this task, students will use unit rates and analyze proportional relationships to calculate average water usage and the minimum amount of water needed for survival each day.
- ▶ **3-Act Mathematical Modeling Lessons** provide real-world tasks that elicit mathematical thinking and engage the Effective Mathematics Teaching Practice “Implement Tasks that Promote Reasoning and Problem Solving” as described in the Teacher’s Edition Program Overview (Grades 6–8). In Grade 7 Topic 2, students use mathematical modeling to represent a problem situation and propose a solution as they use equivalent ratios and proportions to make the same flavor liquid in a glass and in a cooler. Students test and verify the appropriateness of math models and explain why the results from mathematical models may not align exactly to problem situations as they explore possible sources of error inherent in using a math model in a real-world situation. (For example, Grade 7 Teacher’s Edition pp. 111A–114)
- ▶ The **Step 1 Develop: Problem-Based Learning** activity for each lesson, *Solve & Discuss It! Explain It! or Explore It!*, is designed to engage students with a problem in which new math ideas are embedded. Coherence is facilitated as students connect prior knowledge to the new math ideas. Students solve the problem in any way they choose; are given time to struggle; and as students think, conceptual understandings emerge. *Solve & Discuss It!, Explain It! or Explore It!* online utilizes the *DrawPad* to have students write and share their solutions on screen. (For example, Grade 7 Teacher’s Edition Lesson 2-4, p. 103) The questions in the Teacher’s Edition that support the *Solve & Discuss It!, Explain It! or Explore It!* focus specifically on several of the Effective Mathematics Teaching Practices. (For example, Grade 7 Teacher’s Edition Lesson 2-5, p. 115.)
- ▶ **Visual Learning Bridge Step 2 Develop: Visual Learning** of each lesson or the online version, *Visual Learning Animation Plus* and other interactive examples, increase the cognitive level of instruction by connecting concrete and pictorial representations to abstract symbols. The questions provided in the Teacher’s Edition have students reflect on the work that is shown, make connections among ideas, and justify the steps. The *Try It!* and *Convince Me!* features in each lesson foster communication during visual learning. (For example, Grade 7 Teacher’s Edition Lesson 2-4, p. 104)
- ▶ **Practice & Problem Solving** build proficiency as students work on their own. Each Practice & Problem Solving includes a Higher Order Thinking problem. (For example, Grade 7 Teacher’s Edition Lesson 2-5, pp. 119–120)
- ▶ Students communicate often about their thinking and work, using reasoning during **Differentiation Lesson Activities in Part 3** of each lesson through *Enrichment*, *Digital Math Tools Activities*, and online *Math Games*. (For example, Grade 7 Teacher’s Edition Lesson 2-5, p. 120B)

○ Addresses instructional expectations and is easy to understand and use.

- ▶ Clear, in-depth support for Focus, Coherence, and Rigor is found in the **Topic Overview** at the beginning of each topic in the Teacher’s Edition. (For example, Grade 7 Teacher’s Edition pp. 80A–80F)
- ▶ Each **Lesson Overview** provides specific information about Focus, Coherence, and Rigor for the lesson. Online lesson planning provides customization opportunities for each lesson and an online calendar. (For example, Grade 7 Lesson 2-4, p. 103A)

- ▶ The **Listen and Look For Videos** provide specific information about how the lesson’s Essential Understanding is implemented and what the teacher should look for in student work and discussions. (For example, Grade 7 Lesson 2-3, p. 97A)
- ▶ A clear, three-step lesson is provided with comprehensive teaching support for every part of the lesson — both for print and digital pathways.
- ▶ **Problem-Based Learning Solve & Discuss It!, Explain It! or Explore It!** teaching actions are provided with Before, During, and After the activity. Sample student work can be displayed online. Tips for facilitating *Problem-Based Learning* are provided and include:
 - Set Expectations – Make sure students know you expect them to do the thinking.
 - Foster communication – Have students share their thinking with a partner, small group, or the whole class.
 - Be encouraging – Show that you value students’ thinking even when they struggle.
 - Use the language of the math practices during discussions.
- ▶ **Online Visual Learning Animation Plus and Other Interactive Examples** provide direct, stepped-out instruction. The animations, interactivities, and audio enhance learning. *Try It!* and *Convince Me!* are provided to help facilitate class discussion.
- ▶ **Guiding Questions** are provided for a question-driven classroom conversation about each part of the Visual Learning Bridge throughout each lesson.
- ▶ **Math Practices** are highlighted in red throughout the lesson.

○ Provides appropriate level and type of scaffolding, differentiation, intervention and support for a broad range of learners.

- Supports diverse cultural and linguistic backgrounds, interests and styles.
- Provides extra supports for students working below grade level.
- Provides extensions for students with high interest or working above grade level.

▶ **enVisionmath2.0** (Grades 6–8) meets the needs of all students and provides Response to Intervention in the following ways:

▶ **Ongoing Intervention, During the Lesson**

Response to Intervention during the *Visual Learning Bridge* provides additional assistance with one or more of the Examples.

Enrichment during the *Visual Learning Bridge* extends one or more of the Examples to challenge students.

English Language Learners during the *Visual Learning Bridge* develops and reinforces understanding of key terms and concepts.

Prevent Misconceptions during the *Do You Understand?* or *Do You Know How?* sections includes a remediation strategy to address a common misconception about the lesson concept.

Error Intervention during *Practice & Problem Solving* identifies a common error and provides remediation strategy.

Challenge during *Practice & Problem Solving* provides an extension of a problem’s concept to enrich students’ understanding.

Learning Aids Online in *MathXL for School* during the lesson includes personalized practice for *Practice & Problem Solving* for every lesson. *MathXL* is autoscored with on-screen help including *Help Me Solve This* and *View an Example*.

Higher Order Thinking problems in lesson practice challenge student thinking.

▶ **Strategic Intervention, at the End of the Lesson**

Lesson Quiz provides an assessment of the lesson’s concepts and a baseline for prescribing differentiated assignments.

Reteach to Build Understanding provides guided reteaching as a follow-up to the lesson.

Additional Vocabulary Support helps students develop and reinforce understanding of key terms and concepts.

Build Mathematical Literacy provides support for struggling readers to build mathematical literacy.

Digital Math Tools Activities reinforce lesson content or previously taught content.

Online Math Games reinforce lesson content or previously taught content and include thinking games rather than just drill games. Students learn concepts through the games.

Leveled Practice & Problem Solving offers two (or three) pages of homework and practice in the Student's Edition at the end of every lesson. Items focus on skills (often with leveled practice/scaffolding for the first few exercises) and on problem solving that includes reinforcement of math practices, vocabulary, *Higher Order Thinking*, and *Common Core Assessment*.

Additional Practice provides two additional pages of Practice & Problem Solving.

Learning Aids Online in *MathXL for School* include personalized practice for every lesson. *MathXL* is autoscored with on-screen help to include *Help Me Solve This* and *View an Example*.

▶ **Intensive Intervention as Needed Anytime**

Math Diagnosis and Intervention System 2.0 Intervention Lessons contain 2 pages of guided instruction and practice.

Visual Learning Animation Plus is a step-by-step visual learning, interactive animation that is used in each lesson to connect the *Problem-Based Learning Solve & Discuss It, Explain It! or Explore It!* to the lesson concepts in a very visual way. This animation, along with the other interactive examples, can be used anytime to refresh understanding.

Online Math Games can be used anytime for more reinforcement.

Learning Aids Online MathXL for School (described earlier)

ExamView CD-ROM provides unlimited practice exercises for additional practice.

▶ **Math Diagnosis and Intervention System 2.0 (MDIS)** – Helps teachers diagnose students' needs and provides effective intervention that is on or below grade level.

Diagnosis Use the diagnostic tests in the system. Also, use the item analysis charts given with program assessments at the start of a grade or topic, or at the end of a topic, group of topics, or the year.

Intervention Lessons These two-page lessons include guided instruction followed by practice. You can assign lessons that are below grade level if needed.

Teacher Support Teacher Notes provide the support needed to conduct a short lesson. The lesson focuses on vocabulary, concept development, and practice. The Teacher's Guide contains individual and class record forms and correlations to Student's Edition lessons.

▶ **PearsonRealize.com Auto Assigned Differentiation**

Differentiation Before a Topic is based on the result of *Topic Readiness Tests*. Students may be assigned intervention lessons, which include lesson Journal PDFs and MathXL Practice.

Differentiation After a Lesson is based on results of the *Lesson Quiz*. Students are given the Intervention or Advanced assignment based on student performance. The Intervention assignment includes a Reteach to Build Understanding master while the Advanced assignment includes an Enrichment master.

Differentiation After a Group of Lessons is based on the results of the *Mid-Topic Assessment*. Students may be assigned to review specific lesson concepts digitally or to view the Virtual Nerd instructional tutorials.

Differentiation After a Topic or Group of Topics is based on results of online *Topic Assessments* and online *Cumulative/Benchmark Assessments*. Students are assigned remediation which includes digital review of specific lesson concepts and Intervention Lessons in the *Math Diagnosis and Intervention System 2.0*. At any time, a teacher may assign students digital math games or Digital Math Tools Activities as needed for differentiation.

Assignment Reports show the status of assigned resources.

Usage Data lets teachers know how much time students are spending in the online resources.

▶ **Today's Challenge Online** shows 5 problems using the same data on 5 different days. Problems apply prior knowledge and reinforce the kind of thinking students need for success on high-stakes tests. Problems increase in difficulty within a set. *Today's Challenge Teacher's Guide* includes teaching actions organized under Before, During, and After in addition to Vocabulary Review, ELL Support, and Extension.

▶ **English Language Learners** daily ELL instruction support is provided in the Teacher's Edition for every lesson. This support is used with a specified part of the lesson such as such as *Solve & Discuss It!, Explain It!, or Explore It!, Visual Learning Bridge, Try It!, or Convince Me!*. Leveled instruction includes suggestions for students at Beginning, Intermediate, and Advanced levels of English Language Proficiency. ELL Toolkit provides additional support for English Language Learners.

▶ **Visual Learning Animation Plus and Other Interactive Examples Online** provide motion and sound to help lower language barriers to learning. Questions that are read aloud also appear on screen to help English Language Learners connect oral and written language.

- ▶ **Visual Learning Bridge** often has visual models to help give meaning to math language. Instruction is stepped out to visually organize important ideas. Teachers may want to use the Student's Edition eText to display the *Visual Learning Bridge* after using the *Visual Learning Animation Plus* and other interactive examples.
- ▶ **Online Glossary** is always available to students and teachers while using digital resources. The glossary is in English and Spanish to help students connect Spanish math terms they may know to English equivalents.
- ▶ **Pictures with a purpose** that appear in lesson practice provide comprehensible input and help communicate information related to math concepts or to real-world problems.

A unit or longer lesson should:

- Recommend and facilitate a mix of instructional approaches for a variety of learners such as using multiple representations (e.g., including models, using a range of questions, checking for understanding, flexible grouping, pair-share).
 - ▶ Students frequently use models to represent their thinking and solutions in the **Develop: Problem-Based Learning** Part 1 of each lesson, **Solve & Discuss It!, Explain It!, or Explore It!**. Teachers are provided with guiding questions to facilitate this discussion and **Before, During, and After** teaching actions model classroom discourse.
 - Before** uses Teaching Actions #1 (Implement Tasks that Promote Reasoning and Problem Solving) and #2 (Build Understanding) to start understanding. This is whole-class discussion.
 - During** (#3 Support Productive Struggle in Learning Mathematics) is used when students are stumped and are working together in pairs or small groups as the teacher facilitates.
 - After** represents another whole-class discussion. Use teaching action #4 (Facilitate Meaningful Mathematical Discourse) and #5 (Transition to Visual Learning) to discuss students' thinking and work, and to make math ideas explicit. Use #6 (Extension for Early Finishers) as needed.
 - ▶ Blue guiding questions are provided throughout each lesson to support classroom discourse.
 - ▶ *Try It!* and *Convince Me!* checks for understanding right after the first example and connects to the essential understanding of the lesson and to math practices.
 - ▶ A variety of activities are provided for differentiation where students may be working with the teacher in small groups or in pairs on a digital device or in print. Different learning modalities offer greater access for all students.
- Gradually remove supports, requiring students to demonstrate their mathematical understanding independently.
 - ▶ The **enVisionmath2.0** (Grades 6–8) instructional lesson design (print and online) represents a gradual release model.
 - Step 1: Develop: Problem-Based Learning, Solve & Discuss It!, Explain It! or Explore It!*, introduces the lesson by engaging students with a problem in which new math ideas are embedded. Students work in small groups to solve the problem with teacher-facilitated discussion.
 - Step 2: Develop: Visual Learning* is provided whole class through direct instruction from the *Visual Learning Bridge* and/or *Visual Learning Animation Plus* and other interactive examples online where learning is stepped out. *Do You Understand? Do You Know How?* includes teacher-supported concepts and skills practice. *Practice & Problem Solving* builds proficiency as students work on their own.
 - Step 3: Assess & Differentiate* provides additional independent work through *Additional Practice* — two additional pages of homework and practice.
 - ▶ *Practice & Problem Solving* items focus on skills (often with leveled practice/scaffolding for the first few exercises) and on problem solving that includes reinforcement of math practices, vocabulary, *Higher Order Thinking*, and *Common Core Assessment*.
 - ▶ *Additional Practice* items focus on skills (often with scaffolding) and on problem solving that includes reinforcement of math practices, vocabulary, *Higher Order Thinking*, and *Common Core Assessment*.
 - ▶ *Reteach to Build Understanding* provides scaffolded, guided reteaching that prepares students for the homework.
- Demonstrate an effective sequence and a progression of learning where the concepts or skills advance and deepen over time.

- ▶ One or more topics in **enVisionmath2.0** (Grades K–8) focus on each Common Core cluster. Grade 7 Topics 2 and 3 focus on Major Cluster 7.RP.A: Analyze proportional relationships and use them to solve real-world and mathematical problems. Grade 7 Topic 2, Analyze and Use Proportional Relationships, focuses on the deep understanding of ratios, rates, and proportional reasoning. Topic 3, Analyze and Solve Percent Problems, then builds on and extends that understanding to percents.
- ▶ At the start of a topic, one or more *Essential Questions* help students focus on key ideas in the topic. Grade 7 Topic 2 *Essential Question* is, “How can you recognize and represent proportional relationships and use them to solve problems?” Topic 3 *Essential Question* is, “How can percents show proportional relationships between quantities and be used to solve problems?”
- ▶ An *Essential Understanding* is stated in the Teacher’s Edition for each lesson. The Essential Understandings build from each other and show the progression of concepts within a topic.
- ▶ Some of the elements of a lesson that help teachers focus students’ attention on important lesson ideas include the *Lesson Essential Question*, the *Visual Learning Bridge* and its digital counterparts, the *Visual Learning Animation Plus* and other interactive examples, the *Key Concept*, as well as the *Try It!* or *Convince Me!*.

○ Expect, support and provide guidelines for procedural skill and fluency with core calculations and mathematical procedures (when called for in the standards for the grade) to be performed quickly and accurately.

- ▶ **enVisionmath2.0 (Grades K–8) provides steps to fluency success.** Students achieve fluency when they demonstrate skill in carrying out procedures flexibly, accurately, efficiently, and appropriately. Follow the *Steps to Fluency Success* to help all students achieve fluency standard expectations for each grade.

Step 1: Fluency Development with Understanding For each fluency standard, there is a topic that develops fluency with understanding. That topic is the culmination of foundations for fluency taught previously.

Step 2: Ongoing Assessment of Fluency Subskills After Step 1, assess fluency subskills using one of the following:

Fluency Practice/Assessment Worksheets (Grades K–5) in Teacher’s Resource Masters

Fluency Practice/Assessment Worksheets (Grades K–5) generated by ExamView® CD-ROM

Practice Buddy Online (Grades 3–5) or **MathXL for School** (Grades 6–8) at *PearsonRealize.com*. After Step 2, for students who are fluent, go to Step 5.

Step 3: Fluency Intervention For students who struggle with a fluency standard, use one of the following:

MDIS (Math Diagnosis and Intervention System) Lessons for remediation of subskills. You can also give an *MDIS Diagnostic Test* to assess a need for *MDIS lessons* on prerequisite skills.

Practice Buddy Online (Grades 3–5) or **MathXL for School** (Grades 6–8) with intervention also available via the learning aids “*Help Me Solve This*” and “*View an Example.*”

Step 4: Practice on Fluency Subskills After Step 3, give students practice using one of the following. Then reassess (Step 2).

Fluency Practice/Assessment Worksheets (Grades K–5) in Teacher’s Resource Masters

Fluency Practice/Assessment Worksheets (Grades K–5) generated by ExamView® CD-ROM

Practice Buddy Online (Grades 3–5) or **MathXL for School** (Grades 6–8)

Step 5: Fluency Maintenance For fluency maintenance, use some of the following:

Fluency Practice Activities in the Student’s Edition. Students can collaborate, allowing them to help each other as needed.

Fluency Practice/Assessment Worksheets (Grades K–5) in Teacher’s Resource Masters

Fluency Practice/Assessment Worksheets (Grades K–5) generated by ExamView® CD-ROM

Practice Buddy Online (Grades 3–5) or **MathXL for School** (Grades 6–8)

Game Center Online at *PearsonRealize.com*

Step 6: Summative Fluency Assessment For a summative fluency assessment of all students, use one of the following:

Fluency Practice/Assessment Worksheets (Grades K–5) in Teacher’s Resource Masters

Fluency Practice/Assessment Worksheets (Grades K–5) generated by ExamView® CD-ROM

Practice Buddy Online (Grades 3–5) or **MathXL for School** (Grades 6–8)

IV. Assessment

The lesson/unit regularly assesses whether students are mastering standards-based content and skills:

- Is designed to elicit direct, observable evidence of the degree to which a student can independently demonstrate the targeted CCSS.
- Assesses student proficiency using methods that are accessible and unbiased, including the use of grade-level language in student prompts.
- Includes aligned rubrics, answer keys and scoring guidelines that provide sufficient guidance for interpreting student performance.

▶ **enVisionmath2.0** (Grades K–8) provides opportunities for students to independently demonstrate their understanding.

Online assessments can be customized as needed.

Math Practices Proficiency Rubric is provided and can be used at any time.

Scoring rubrics and sample student work are included for some assessments in the *Assessment Sourcebook*.

▶ The formats of the assessment items, which prepare students for high-stakes Common Core tests, include the following:

Selected response, e.g., single response, multiple response

Constructed response, e.g., short or extended responses, sometimes using an on-screen symbols palette

Technology-enhanced items, e.g., drag and drop, drop-down menus, graphing, on-screen tools

Performance tasks, hand scored or machine scored

▶ A variety of auto-generated assessment reports are available for online assessments.

Individual and class views of progress are provided in an easy-to-view format.

Common Core Standards mastery reports show individual students' mastery or class-wide mastery for each standard.

▶ **Assessment data** can be used to organize students into groups for purposes of making instructional decisions and assigning differentiation resources.

▶ Every lesson includes **Common Core practice items** in formats that help prepare students for Common Core high-stakes tests. These types of items are also included in Readiness Assessments, Mid-topic Checkpoints and Performance Tasks, Lesson Quizzes, End of Topic Assessments and Performance Tasks.

A unit or longer lesson should:

- Use varied modes of curriculum-embedded assessments that may include pre-, formative, summative and self-assessment measures.

▶ **enVisionmath2.0** (Grades K–8) provides students with multiple ways to show what they have learned.

▶ **DIAGNOSTIC ASSESSMENT**

At the start of the YEAR

Beginning-of-Year Assessment Online in TestNav™

Beginning-of-Year Assessment Masters in Assessment Sourcebook

Diagnostic Test Masters in Math Diagnosis and Intervention System 2.0

At the start of a TOPIC

Topic Readiness Assessment Online in TestNav™

Topic Readiness Assessment Masters in Assessment Sourcebook

Diagnostic Test Masters in Math Diagnosis and Intervention System 2.0

Review What You Know in Student's Edition or eText

▶ **FORMATIVE ASSESSMENT**

During a LESSON

Questions in the *Visual Learning Animation Plus* and *Other Interactive Examples*

Questions to use with the *Visual Learning Bridge* in Teacher's Edition

Try It! and *Convince Me!* in Student's Edition or eText

Do You Understand? Do you Know How? in Student's Edition or eText

At the end of a LESSON

Lesson Quiz Online in TestNav™

Lesson Quiz Masters in Assessment Sourcebook

▶ **SUMMATIVE ASSESSMENT**

In the middle of a TOPIC

Mid-Topic Checkpoint in Student's Edition or eText

Mid-Topic Assessment Online in TestNav™

Mid-Topic Assessment Masters in *Assessment Sourcebook*

Self-Assessment Tool in *Mid-Topic Assessment*

Mid-Topic Performance Task in Student's Edition or eText

At the end of a TOPIC

Topic Assessment Masters in *Assessment Sourcebook*

Topic Assessment Online in TestNav™

Topic Assessment by ExamView® CD-ROM

Topic Performance Assessment Masters in *Assessment Sourcebook*

Fluency Assessment by Practice Buddy Online (Grades K–5)

Fluency Practice/Assessment Masters in *Teacher's Resource Masters* (Grades K–5)

Basic-Facts Timed Test Masters (Grades 1–5) in *Assessment Sourcebook*

After a group of TOPICS

Cumulative/Benchmark Assessment Online in TestNav™

Cumulative/Benchmark Assessment Masters in *Assessment Sourcebook*

Practice Performance Tasks Assessment Online in TestNav™

Practice Performance Tasks Assessment Masters in *Assessment Sourcebook*

At the end of the YEAR

End-of-Year Assessment Online in TestNav™

End-of-Year Assessment Masters in *Assessment Sourcebook Masters*

Next Generation Assessment Practice Test Online in TestNav™

Next Generation Assessment Practice Test Assessment Masters in *Assessment Sourcebook*

▶ **SELF ASSESSMENT**

Self-Assessment Tool in *Teaching Tools* section of the *Teacher's Resource Masters* (Grades K–8)

Self-Assessment Tool in *Mid-Topic Assessment* in *Assessment Sourcebook*

The EQuIP rubric is derived from the Tri-State Rubric and the collaborative development process led by Massachusetts, New York, and Rhode Island and facilitated by Achieve.

This version of the EQuIP rubric is current as of 06-15-13.

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