enVisionMATH™

Grades K-6

Correlated to the Common Core State Standards

PEARSON

ALWAYS LEARNING

Common Core State Standards
Standards for Mathematical Practice
Kindergarten

The Standards for Mathematical Practice are an important part of the Common Core State Standards. They describe varieties of proficiency that teachers should focus on helping in their students develop. These practices draw from the NCTM process standards of problem solving, reasoning and proof, communication, representation, and connections and the strands of mathematical proficiency specified in the National Research Council's report *Adding It Up*: adaptive reasoning, strategic competence, conceptual understanding, procedural fluency, and productive disposition.

For each of the Standards for Mathematical Practice is a explanation of the different features and elements of *Scott Foresman • Addison Wesley enVisionMATH®* that help students develop mathematical proficiency.

1. Make sense of problems and persevere in solving them.

Mathematically proficient students start by explaining to themselves the meaning of a problem and looking for entry points to its solution. They analyze givens, constraints, relationships, and goals. They make conjectures about the form and meaning of the solution and plan a solution pathway rather than simply jumping into a solution attempt. They consider analogous problems, and try special cases and simpler forms of the original problem in order to gain insight into its solution. They monitor and evaluate their progress and change course if necessary. Older students might, depending on the context of the problem, transform algebraic expressions or change the viewing window on their graphing calculator to get the information they need. Mathematically proficient students can explain correspondences between equations, verbal descriptions, tables, and graphs or draw diagrams of important features and relationships, graph data, and search for regularity or trends. Younger students might rely on using concrete objects or pictures to help conceptualize and solve a problem. Mathematically proficient students check their answers to problems using a different method, and they continually ask themselves, "Does this make sense? "They can understand the approaches of others to solving complex problems and identify correspondences between different approaches.

Scott Foresman • Addison Wesley enVisionMATH is built on a foundation of problem-based instruction that helps students develop strong problem-solving skills. Every lesson begins with Problem-based Interactive Learning in which children work their classmates and teachers to make sense of problems and to develop a plan to solve the problem presented. With the problem solving lessons in every topic, children focus on the problem-solving process and strengthen their sense-making skills. The Quick Check provides daily opportunities for children to demonstrate problem-solving skills and strategies.

Throughout the program; for examples, see *enVisionMATH* Kindergarten Lessons 1-5, 2-6, 3-5, 4-10, 5-11, 6-5, 7-9, 8-3, 8-6, 9-5, 9-10, 10-7, 11-7, 12-10, 13-6, 14-7, 15-7, 16-7

2 Reason abstractly and quantitatively.

Mathematically proficient students make sense of quantities and their relationships in problem situations. They bring two complementary abilities to bear on problems involving quantitative relationships: the ability to decontextualize—to abstract a given situation and represent it symbolically and manipulate the representing symbols as if they have a life of their own, without necessarily attending to their referents—and the ability to contextualize, to pause as needed during the manipulation process in order to probe into the referents for the symbols involved. Quantitative reasoning entails habits of creating a coherent representation of the problem at hand; considering the units involved; attending to the meaning of quantities, not just how to compute them; and knowing and flexibly using different properties of operations and objects.

Reasoning is another important theme of the Scott Foresman • Addison Wesley enVisionMATH Program. In most lessons, the Visual Learning Bridge presents a situation and students are shown how the situation can be represented numerically or algebraically. Later in a lesson, students have opportunities to work on their own to represent situations symbolically. Through the solving process, students are encouraged to think about their solutions and determine whether the solutions they found are reasonable. Often, the Do You Understand questions focus on helping children begin to reason abstractly.

Throughout the program; for examples, see Lessons 1-5, 5-2, 5-4, 5-5, 5-7, 5-8, 12-5, 14-7

3 Construct viable arguments and critique the reasoning of others.

Mathematically proficient students understand and use stated assumptions, definitions, and previously established results in constructing arguments. They make conjectures and build a logical progression of statements to explore the truth of their conjectures. They are able to analyze situations by breaking them into cases, and can recognize and use counterexamples. They justify their conclusions, communicate them to others, and respond to the arguments of others. They reason inductively about data, making plausible arguments that take into account the context from which the data arose. Mathematically proficient students are also able to compare the effectiveness of two plausible arguments, distinguish correct logic or reasoning from that which is flawed, and—if there is a flaw in an argument—explain what it is. Elementary students can construct arguments using concrete referents such as objects, drawings, diagrams, and actions. Such arguments can make sense and be correct, even though they are not generalized or made formal until later grades. Later, students learn to determine domains to which an argument applies. Students at all grades can listen or read the arguments of others, decide whether they make sense, and ask useful questions to clarify or improve the arguments.

Consistent with a focus on reasoning and sense-making is a focus on critical reasoning – argumentation and critique of arguments. In Pearson's *Scott Foresman • Addison Wesley enVisionMATH*, the Problem-Based Interactive Learning affords students opportunities to share with classmates their thinking about problems, their solutions, and their reasoning about the solutions. Articulating clearly an explanation for a process is a stepping stone to critical analysis and reasoning of both their own processes and those of others.

Throughout the program; for examples, see Lessons 1-3, 2-2, 3-6, 7-2, 8-3, 9-9, 14-1, 16-4

4 Model with mathematics.

Mathematically proficient students can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace. In early grades, this might be as simple as writing an addition equation to describe a situation. In middle grades, a student might apply proportional reasoning to plan a school event or analyze a problem in the community. By high school, a student might use geometry to solve a design problem or use a function to describe how one quantity of interest depends on another. Mathematically proficient students who can apply what they know are comfortable making assumptions and approximations to simplify a complicated situation, realizing that these may need revision later. They are able to identify important quantities in a practical situation and map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts and formulas. They can analyze those relationships mathematically to draw conclusions. They routinely interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose.

Students in Pearson's Scott Foresman • Addison Wesley enVisionMATH, are introduced to mathematical modeling in the early grades. They first use manipulatives and drawings and then equations to model addition and subtraction situations. The Visual Learning Bridge and Visual Learning Animation often present real-world situations and students are shown how these can be modeled mathematically. In later years, students expand their modeling skills to include other graphical representations such as tables, graphs, as well as equations.

Throughout the program; for examples, see Lessons 4-7, 5-7, 6-4, 7-8, 8-1, 10-3, 11-5, 12-3

5 Use appropriate tools strategically.

Mathematically proficient students consider the available tools when solving a mathematical problem. These tools might include pencil and paper, concrete models, a ruler, a protractor, a calculator, a spreadsheet, a computer algebra system, a statistical package, or dynamic geometry software. Proficient students are sufficiently familiar with tools appropriate for their grade or course to make sound decisions about when each of these tools might be helpful, recognizing both the insight to be gained and their limitations. For example, mathematically proficient high school students analyze graphs of functions and solutions generated using a graphing calculator. They detect possible errors by strategically using estimation and other mathematical knowledge. When making mathematical models, they know that technology can enable them to visualize the results of varying assumptions, explore consequences, and compare predictions with data. Mathematically proficient students at various grade levels are able to identify relevant external mathematical resources, such as digital content located on a website, and use them to pose or solve problems. They are able to use technological tools to explore and deepen their understanding of concepts.

Students become fluent in the use of a wide assortment of tools ranging from physical objects, including manipulatives, rulers, protractors, and even pencil and paper, to technological tools, such as etools, calculators and computers. As students become more familiar with the tools available to them, they are able to begin making decisions about which tools are more appropriate to solve different kinds of problems.

Throughout the program; for examples, see Lessons 4-5, 6-1, 9-8, 10-5, 11-6, 12-8, 14-5, 15-5

6 Attend to precision.

Mathematically proficient students try to communicate precisely to others. They try to use clear definitions in discussion with others and in their own reasoning. They state the meaning of the symbols they choose, including using the equal sign consistently and appropriately. They are careful about specifying units of measure, and labeling axes to clarify the correspondence with quantities in a problem. They calculate accurately and efficiently, express numerical answers with a degree of precision appropriate for the problem context. In the elementary grades, students give carefully formulated explanations to each other. By the time they reach high school they have learned to examine claims and make explicit use of definitions.

Students are expected to use mathematical terms and symbols with precision. In the early years, as children develop their mathematical vocabulary, they are encouraged to use terms accurately. In later years, key terms and concepts are highlighted in each lesson. In the **Do You Understand** feature, students often revisit these key terms and provide explicit definitions or explanations of the terms. For the Writing to Explain and Think About a Process Exercises, students are to provide clear explanations of terms, concepts, or processes and to use new terms accurately and precisely. Students are reminded to use appropriate units of measure when working through solutions and accurate labels on axes when making graphs to represent solutions.

Throughout the program; for examples, see Lessons 1-2, 7-9, 8-5, 9-5, 14-1, 14-2, 14-7, 16-6

7 Look for and make use of structure.

Mathematically proficient students look closely to discern a pattern or structure .Young students, for example, might notice that three and seven more is the same amount as seven and three more, or they may sort a collection of shapes according to how many sides the shapes have. Later, students will see 7×8 equals the well remembered $7 \times 5 \times 7 \times 3$, in preparation for learning about the distributive property. In the expression $x^2 + 9x + 14$, older students can see the 14 as 2×7 and the 9 as $2 + 7$.They recognize the significance of an existing line in a geometric figure and can use the strategy of drawing an auxiliary line for solving problems. They also can step back for an overview and shift perspective. They can see complicated things, such as some algebraic expressions, as single objects or as being composed of several objects. For example, they can see $5\ \ 3(x\ \ y)^2$ as 5 minus a positive number times a square and use that to realize that its value cannot be more than 5 for any real numbers x and y.

Throughout the program, students are encouraged to look for patterns and structure as they look to develop solution plans. In the *Look for a Pattern* Problem-Solving lessons, children in the early years develop a sense of patterning with visual and physical objects.

As students mature in their mathematical thinking, they look for patterns in numerical operations by focusing on place value and properties of operations. From this focus on looking for and recognizing patterns, students become well-equip to draw from these patterns to formalize their thinking about the structure of operations.

Throughout the program; for examples, see Lessons 3-3, 5 10, 7-1, 9-1, 10-6, 11-6, 12-8, 15-4

8 Look for and express regularity in repeated reasoning.

Mathematically proficient students notice if calculations are repeated, and look both for general methods and for shortcuts. Upper elementary students might notice when dividing 25 by 11 that they are repeating the same calculations over and over again, and conclude they have a repeating decimal. By paying attention to the calculation of slope as they repeatedly check whether points are on the line through (1, 2) with slope 3, middle school students might abstract the equation $(y - 2)/(x - 1) = 3$. Noticing the regularity in the way terms cancel when expanding $(x - 1)(x + 1)$, $(x - 1)(x^2 + x + 1)$, and $(x - 1)(x^3 + x^2 + x + 1)$ might lead them to the general formula for the sum of a geometric series. As they work to solve a problem, mathematically proficient students maintain oversight of the process, while attending to the details. They continually evaluate the reasonableness of their intermediate results.

Once again, throughout the program as a whole, students are prompted to look for repetition in computations to look for shortcuts that can make the problem-solving process more efficient. Students are prompted to think about problems they encountered previously that may share features or processes. They are encouraged to draw on the solution plan developed for that problem, and as their mathematical thinking matures, to look for generalizations that can be applied to other problem situations. The **Problem-Based Interactive Learning** activities offer students opportunities to look for regularity in the way operations behave.

Throughout the program; for examples, see Lessons 1-2, 4-7, 5-2, 8-5, 10-3, 11-2, 12-5, 16-6

Correlation of Standards for Mathematical Content

enVisionMATH® Kindergarten

The following shows the alignment of *enVisionMATH* Kindergarten ©2009/2011 to the Common Core State Standards for Kindergarten. Included in this correlation are the supplemental lessons that will be available as part of the transitional support that Pearson is providing. These lessons will available in the summer 2011.

Standards for Mathematical Content Kindergarten		Where to find in *enVisionMATH* ©2009/2011
Counting and Cardinality		
Know number names and the count sequence.		
K.CC.1	Count to 100 by ones and by tens.	12-6, 12-7, 12-8
K.CC.2	Count forward beginning from a given number within the known sequence (instead of having to begin at 1).	5-10, 12-6, 12-10
K.CC.3	Write numbers from 0 to 20. Represent a number of objects with a written numeral 0–20 (with 0 representing a count of no objects).	4-2, 4-4, 4-5, 5-3, 5-6, 5-9, 12-1, 12-2, 12-3, 12-4
Count to tell the number of objects.		
K.CC.4	Understand the relationship between numbers and quantities; connect counting to cardinality.	4-1, 4-2, 4-3, 4-4, 4-5, 5-1, 5-3, 5-4, 5-6, 5-7, 5-9, 12-1, 12-2, 12-3, 12-4, 12-6
K.CC.4.a	When counting objects, say the number names in the standard order, pairing each object with one and only one number name and each number name with one and only one object.	3-6, 4-1, 4-2, 4-3, 4-4, 4-5, 5-1, 5-3, 5-4, 5-6, 5-7, 5-9, 12-1, 12-2, 12-3, 12-4, 12-6
K.CC.4.b	Understand that the last number name said tells the number of objects counted. The number of objects is the same regardless of their arrangement or the order in which they were counted.	3-6, 4-1, 4-2a, 4-2, 4-3, 4-4a, 4-4, 4-5, 4-10, 5-1, 5-3, 5-4, 5-6, 5-7, 5-9, 12-1, 12-2, 12-3, 12-4, 12-6
K.CC.4.c	Understand that each successive number name refers to a quantity that is one larger.	3-6, 4-1, 4-2, 4-3, 4-4, 4-5, 5-1, 5-3, 5-4, 5-6, 5-7, 5-9, 12-1, 12-2, 12-3, 12-4, 12-6
K.CC.5	Count to answer "how many?" questions about as many as 20 things arranged in a line, a rectangular array, or a circle, or as many as 10 things in a scattered configuration; given a number from 1–20, count out that many objects.	4-1, 4-2a, 4-2, 4-3, 4-4a, 4-4, 4-5, 4-10, 5-1, 5-3, 5-4, 5-6, 5-7, 5-9, 12-1, 12-2, 12-3, 12-4

Standards for Mathematical Content Kindergarten		Where to find in *enVisionMATH* ©2009/2011
Compare numbers.		
K.CC.6	Identify whether the number of objects in one group is greater than, less than, or equal to the number of objects in another group, e.g., by using matching and counting strategies. (Include groups with up to ten objects.)	4-7, 4-8, 4-9, 6-1, 6-2, 6-3, 6-4, 6-5, 16-1
K.CC.7	Compare two numbers between 1 and 10 presented as written numerals.	6-1, 6-2, 6-3, 6-4

Standards for Mathematical Content Kindergarten		Where to find in *enVisionMATH* ©2009/2011
Operations and Algebraic Thinking		
Understand addition as putting together and adding to, and understand subtraction as taking apart and taking from.		
K.OA.1	Represent addition and subtraction with objects, fingers, mental images, drawings, sounds (e.g., claps), acting out situations, verbal explanations, expressions, or equations. (Drawings need not show details, but should show the mathematics in the problem. (This applies wherever drawings are mentioned in the Standards.))	2-6, 6-4, 10-1, 10-2, 10-3, 10-4, 10-5, 10-6, 10-7, 11-1, 11-2, 11-3, 11-4, 11-5, 11-6, 11-7
K.OA.2	Solve addition and subtraction word problems, and add and subtract within 10, e.g., by using objects or drawings to represent the problem.	2-6, 10-1, 10-2, 10-3, 10-4, 10-5, 10-6, 10-7, 11-1, 11-2, 11-3, 11-4, 11-5, 11-6, 11-7
K.OA.3	Decompose numbers less than or equal to 10 into pairs in more than one way, e.g., by using objects or drawings, and record each decomposition by a drawing or equation (e.g., 5 = 2 + 3 and 5 = 4 + 1).	4-6, 4-7a, 5-2, 5-4a, 5-5, 5-7a, 5-8, 5-10a
K.OA.4	For any number from 1 to 9, find the number that makes 10 when added to the given number, e.g., by using objects or drawings, and record the answer with a drawing or equation.	5-8
K.OA.5	Fluently add and subtract within 5.	10-1, 10-2, 10-3, 10-4, 10-5, 10-6, 10-7, 11-1, 11-2, 11-3, 11-4, 11-5, 11-6, 11-7

Standards for Mathematical Content Kindergarten		Where to find in *enVisionMATH* ©2009/2011
Number and Operations in Base Ten		
Work with numbers 11–19 to gain foundations for place value.		
K.NBT.1	Compose and decompose numbers from 11 to 19 into ten ones and some further ones, e.g., by using objects or drawings, and record each composition or decomposition by a drawing or equation (e.g., 18 = 10 + 8); understand that these numbers are composed of ten ones and one, two, three, four, five, six, seven, eight, or nine ones.	12-3a, 12-4a, 12-5a 12-5b, 12-5c, 12-5d, 12-5e

Standards for Mathematical Content Kindergarten		Where to find in *enVisionMATH* ©2009/2011
Measurement and Data		
Describe and compare measurable attributes.		
K.MD.1	Describe measurable attributes of objects, such as length or weight. Describe several measurable attributes of a single object.	9-1, 9-2, 9-3, 9-4, 9-5, 9-6, 9-7, 9-8, 9-9, 9-10
K.MD.2	Directly compare two objects with a measurable attribute in common, to see which object has "more of"/"less of" the attribute, and describe the difference.	9-1, 9-2, 9-3, 9-5, 9-6, 9-8
Classify objects and count the number of objects in each category.		
K.MD.3	Classify objects into given categories; count the numbers of objects in each category and sort the categories by count. (Limit category counts to be less than or equal to 10.)	1-1, 1-2, 1-3, 1-4, 1-5, 5-11, 16-3, 16-4, 16-5, 16-7

Standards for Mathematical Content Kindergarten		Where to find in *enVisionMATH* ©2009/2011
Geometry		
Identify and describe shapes (squares, circles, triangles, rectangles, hexagons, cubes, cones, cylinders, and spheres).		
K.G.1	Describe objects in the environment using names of shapes, and describe the relative positions of these objects using terms such as above, below, beside, in front of, behind, and next to. [Note: Students do not use words to describe positions.]	1-5, 2-1, 2-2, 2-3, 2-4, 2-5, 2-6, 7-1, 7-2, 7-6
K.G.2	Correctly name shapes regardless of their orientations or overall size.	7-1, 7-2, 7-4, 7-6, 7-7a
K.G.3	3. Identify shapes as two-dimensional (lying in a plane, "flat") or three-dimensional ("solid").	7-1, 7-2, 7-6, 7-8
Analyze, compare, create, and compose shapes.		
K.G.4	Analyze and compare two- and three-dimensional shapes, in different sizes and orientations, using informal language to describe their similarities, differences, parts (e.g., number of sides and vertices/"corners") and other attributes (e.g., having sides of equal length).	7-1, 7-2, 7-4a, 7-7a, 7-7, 7-8
K.G.5	Model shapes in the world by building shapes from components (e.g., sticks and clay balls) and drawing shapes.	7-8
K.G.6	Compose simple shapes to form larger shapes. For example, "Can you join these two triangles with full sides touching to make a rectangle?"	7-3, 7-4a

Common Core State Standards

Standards for Mathematical Practices
Grade 1

The Standards for Mathematical Practice are an important part of the Common Core State Standards. They describe varieties of proficiency that teachers should focus on developing in their students. These practices draw from the NCTM process standards of problem solving, reasoning and proof, communication, representation, and connections and the strands of mathematical proficiency specified in the National Research Council's report *Adding It Up*: adaptive reasoning, strategic competence, conceptual understanding, procedural fluency, and productive disposition.

For each of the Standards for Mathematical Practices is an explanation of the different features and elements of *Scott Foresman • Addison Wesley enVisionMATH*TM that help students develop mathematical proficiency.

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1. Make sense of problems and persevere in solving them.

Mathematically proficient students start by explaining to themselves the meaning of a problem and looking for entry points to its solution. They analyze givens, constraints, relationships, and goals. They make conjectures about the form and meaning of the solution and plan a solution pathway rather than simply jumping into a solution attempt. They consider analogous problems, and try special cases and simpler forms of the original problem in order to gain insight into its solution. They monitor and evaluate their progress and change course if necessary. Older students might, depending on the context of the problem, transform algebraic expressions or change the viewing window on their graphing calculator to get the information they need. Mathematically proficient students can explain correspondences between equations, verbal descriptions, tables, and graphs or draw diagrams of important features and relationships, graph data, and search for regularity or trends. Younger students might rely on using concrete objects or pictures to help conceptualize and solve a problem. Mathematically proficient students check their answers to problems using a different method, and they continually ask themselves, "Does this make sense? "They can understand the approaches of others to solving complex problems and identify correspondences between different approaches.

Scott Foresman • Addison Wesley enVisionMATH is built on a foundation of problem-based instruction that has sense-making at its heart. The structure of each lesson facilitates students' implementation of this process. Every lesson begins with Problem-Based Interactive Learning, an activity in which students are presented a problem to solve. They interact with their peers and teachers to make sense of the problem presented and to look for a workable solution. A second feature of each lesson are the Problem Solving Exercises for which students persevere to find solutions for each exercise. In each topic is at least one Problem Solving lesson with a primary focus of honing students' sense-making and problem-solving skills.

Throughout the program; for examples, see Lessons 1-9, 2-11, 3-5, 4-10, 6-7, 7-8, 8-6, 9-3, 11-5, 11-6, 12-7, 13-4, 14-7, 16-4

2 Reason abstractly and quantitatively.

Mathematically proficient students make sense of quantities and their relationships in problem situations. They bring two complementary abilities to bear on problems involving quantitative relationships: the ability to decontextualize—to abstract a given situation and represent it symbolically and manipulate the representing symbols as if they have a life of their own, without necessarily attending to their referents—and the ability to contextualize, to pause as needed during the manipulation process in order to probe into the referents for the symbols involved. Quantitative reasoning entails habits of creating a coherent representation of the problem at hand; considering the units involved; attending to the meaning of quantities, not just how to compute them; and knowing and flexibly using different properties of operations and objects.

Reasoning is another important theme of the *Scott Foresman • Addison Wesley enVisionMATH* Program. In most lessons, the Visual Learning Bridge presents a situation and students are shown how the situation can be represented numerically or algebraically. Later in a lesson, students have opportunities to work on their own to represent situations symbolically. Through the solving process, students become aware of the importance of checking problem solutions with the Reasonableness exercises. In the Do You Understand part of the Guided Practice, students are often asked to consider the meaning of different parts of an expression or equation. The Journal Exercises also help students to be thinking and reasoning abstractly and quantitatively.

Throughout the program; for examples, see Lessons 2-3, 2-9, 3-3, 6-7, 7-3, 7-7, 8-3, 9-3, 12-7, 14-4, 15-8

3 Construct viable arguments and critique the reasoning of others.

Mathematically proficient students understand and use stated assumptions, definitions, and previously established results in constructing arguments. They make conjectures and build a logical progression of statements to explore the truth of their conjectures. They are able to analyze situations by breaking them into cases, and can recognize and use counterexamples. They justify their conclusions, communicate them to others, and respond to the arguments of others. They reason inductively about data, making plausible arguments that take into account the context from which the data arose. Mathematically proficient students are also able to compare the effectiveness of two plausible arguments, distinguish correct logic or reasoning from that which is flawed, and—if there is a flaw in an argument—explain what it is. Elementary students can construct arguments using concrete referents such as objects, drawings, diagrams, and actions. Such arguments can make sense and be correct, even though they are not generalized or made formal until later grades. Later, students learn to determine domains to which an argument applies. Students at all grades can listen or read the arguments of others, decide whether they make sense, and ask useful questions to clarify or improve the arguments.

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Throughout the program; for examples, see Lessons 2-3, 4-7, 7-4, 8-1, 8-7, 12-3, 14-3

4 Model with mathematics.

Mathematically proficient students can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace. In early grades, this might be as simple as writing an addition equation to describe a situation. In middle grades, a student might apply proportional reasoning to plan a school event or analyze a problem in the community. By high school, a student might use geometry to solve a design problem or use a function to describe how one quantity of interest depends on another. Mathematically proficient students who can apply what they know are comfortable making assumptions and approximations to simplify a complicated situation, realizing that these may need revision later. They are able to identify important quantities in a practical situation and map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts and formulas. They can analyze those relationships mathematically to draw conclusions. They routinely interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose.

Students in Pearson's *Scott Foresman • Addison Wesley enVisionMATH,* are introduced to mathematical modeling in the early grades. They first use manipulatives and drawings and then equations to model addition and subtraction situations. The Visual Learning Bridge and Visual Learning Animation often present real-world situations and students are shown how these can be modeled mathematically. In later years, students expand their modeling skills to include other graphical representations such as tables, graphs, as well as equations.

Throughout the program; for examples, see Lessons 1-5, 2-1, 1-9, 4-4, 5-2, 6-6, 7-4, 8-9, 9-2, 10-1, 11-4, 12-5, 16-2

5 Use appropriate tools strategically.

Mathematically proficient students consider the available tools when solving a mathematical problem. These tools might include pencil and paper, concrete models, a ruler, a protractor, a calculator, a spreadsheet, a computer algebra system, a statistical package, or dynamic geometry software. Proficient students are sufficiently familiar with tools appropriate for their grade or course to make sound decisions about when each of these tools might be helpful, recognizing both the insight to be gained and their limitations. For example, mathematically proficient high school students analyze graphs of functions and solutions generated using a graphing calculator. They detect possible errors by strategically using estimation and other mathematical knowledge. When making mathematical models, they know that technology can enable them to visualize the results of varying assumptions, explore consequences, and compare predictions with data. Mathematically proficient students at various grade levels are able to identify relevant external mathematical resources, such as digital content located on a website, and use them to pose or solve problems. They are able to use technological tools to explore and deepen their understanding of concepts.

Students become fluent in the use of a wide assortment of tools ranging from physical objects, including manipulatives, rulers, protractors, and even pencil and paper, to technological tools, such as etools, calculators and computers. As students become more familiar with the tools available to them, they are able to begin making decisions about which tools are more appropriate to solve different kinds of problems.

Throughout the program; for examples, see Lessons 8-6, 14-4, 14-5, 14-8, 14-11, 15-3

6 Attend to precision.

Mathematically proficient students try to communicate precisely to others. They try to use clear definitions in discussion with others and in their own reasoning. They state the meaning of the symbols they choose, including using the equal sign consistently and appropriately. They are careful about specifying units of measure, and labeling axes to clarify the correspondence with quantities in a problem. They calculate accurately and efficiently, express numerical answers with a degree of precision appropriate for the problem context. In the elementary grades, students give carefully formulated explanations to each other. By the time they reach high school they have learned to examine claims and make explicit use of definitions.

Students are expected to use mathematical terms and symbols with precision. Key terms and concepts are highlighted in each lesson. In the **Do You Understand** feature, students often revisit these key terms and provide explicit definitions or explanations of the terms. For the *Journal* Exercises, students are to provide clear explanations of terms, concepts, or processes and to use new terms accurately and precisely.

Throughout the program; for examples, see Lessons 1-3, 3-2, 7-3, 8-2, 11-1, 15-3, 16-7

7 Look for and make use of structure.

Mathematically proficient students look closely to discern a pattern or structure .Young students, for example, might notice that three and seven more is the same amount as seven and three more, or they may sort a collection of shapes according to how many sides the shapes have. Later, students will see 7×8 equals the well remembered $7 \times 5 \times 7 \times 3$, in preparation for learning about the distributive property. In the expression $x^2 + 9x + 14$, older students can see the 14 as 2×7 and the 9 as $2 + 7$.They recognize the significance of an existing line in a geometric figure and can use the strategy of drawing an auxiliary line for solving problems. They also can step back for an overview and shift perspective. They can see complicated things, such as some algebraic expressions, as single objects or as being composed of several objects. For example, they can see $5\ \ 3(x\ \ y)^2$ as 5 minus a positive number times a square and use that to realize that its value cannot be more than 5 for any real numbers x and y.

Throughout the program, students are encouraged to look for patterns and structure as they look to develop solution plans. In the *Look for a Pattern* Problem-Solving lessons, children in the early years develop a sense of patterning with visual and physical objects.

As students mature in their mathematical thinking, they look for patterns in numerical operations by focusing on place value and properties of operations. From this focus on looking for and recognizing patterns, students become well-equip to draw from these patterns to formalize their thinking about the structure of operations.

Throughout the program; for examples, see Lessons 1-4, 5-1, 6-3, 7-4, 10-6, 12-3, 16-1

8 Look for and express regularity in repeated reasoning.

Mathematically proficient students notice if calculations are repeated, and look both for general methods and for shortcuts. Upper elementary students might notice when dividing 25 by 11 that they are repeating the same calculations over and over again, and conclude they have a repeating decimal. By paying attention to the calculation of slope as they repeatedly check whether points are on the line through (1, 2) with slope 3, middle school students might abstract the equation $(y - 2)/(x - 1) = 3$. Noticing the regularity in the way terms cancel when expanding $(x - 1)(x + 1)$, $(x - 1)(x^2 + x + 1)$, and $(x - 1)(x^3 + x^2 + x + 1)$ might lead them to the general formula for the sum of a geometric series. As they work to solve a problem, mathematically proficient students maintain oversight of the process, while attending to the details. They continually evaluate the reasonableness of their intermediate results.

Once again, throughout the program as a whole, students are prompted to look for repetition in computations to look for shortcuts that can make the problem-solving process more efficient. Students are prompted to think about problems they encountered previously that may share features or processes. They are encouraged to draw on the solution plan developed for that problem, and as their mathematical thinking matures, to look for generalizations that can be applied to other problem situations. The **Problem-Based Interactive Learning** activities offer students opportunities to look for regularity in the way operations behave.

Throughout the program; for examples, see Lessons 3-6, 8-7, 13-5, 16-7

Correlation of Standards for Mathematical Content

enVisionMATH™ Grade 1

The following shows the alignment of *enVisionMATH* Grade 1 ©2009/2011 to the Common Core State Standards for Grade 1. Included in this correlation are the supplemental lessons that will be available as part of the transitional support that Pearson is providing. These lessons will be part of the Transition Kit 2.0, available in May 2011.

Standards for Mathematical Content Grade 1		Where to find in *enVisionMATH* ©2009/2011
Operations and Algebraic Thinking		
Represent and solve problems involving addition and subtraction.		
1.OA.1	Use addition and subtraction within 20 to solve word problems involving situations of adding to, taking from, putting together, taking apart, and comparing, with unknowns in all positions, e.g., by using objects, drawings, and equations with a symbol for the unknown number to represent the problem.	3-1, 3-2, 3-3, 3-4, 3-5, 3-7, 4-1, 4-2, 4-3, 4-4, 4-5, 4-6, 4-7, 4-8, 6-6, 7-1, 7-2, 7-3, 7-4, 7-5, 16-1, 16-2, 16-3, 16-4, 16-5, 16-6, 17-5, CC-1, CC-2
1.OA.2	Solve word problems that call for addition of three whole numbers whose sum is less than or equal to 20, e.g., by using objects, drawings, and equations with a symbol for the unknown number to represent the problem.	16-7, CC-8
Understand and apply properties of operations and the relationship between addition and subtraction.		
1.OA.3	Apply properties of operations as strategies to add and subtract. Examples: If 8 + 3 = 11 is known, then 3 + 8 = 11 is also known. (Commutative property of addition.) To add 2 + 6 + 4, the second two numbers can be added to make a ten, so 2 + 6 + 4 = 2 + 10 = 12. (Associative property of addition.) (Students need not use formal terms for these properties.)	3-6, 6-1, 16-7, CC-8
1.OA.4	Understand subtraction as an unknown-addend problem. For example, subtract 10 – 8 by finding the number that makes 10 when added to 8.	4-1, 4-2, 4-3, 4-4, 4-5, 4-6, 4-7, 5-4, 7-2, 7-3, 7-4, 17-2, 17-3, 17-4, CC-2
Add and subtract within 20.		
1.OA.5	Relate counting to addition and subtraction (e.g., by counting on 2 to add 2).	6-1, 7-1
1.OA.6	Add and subtract within 20, demonstrating fluency for addition	4-1, 4-2, 4-3, 4-4, 4-5,

Standards for Mathematical Content Grade 1		Where to find in *enVisionMATH* ©2009/2011
	and subtraction within 10. Use strategies such as counting on; making ten (e.g., 8 + 6 = 8 + 2 + 4 = 10 + 4 = 14); decomposing a number leading to a ten (e.g., 13 – 4 = 13 – 3 – 1 = 10 – 1 = 9); using the relationship between addition and subtraction (e.g., knowing that 8 + 4 = 12, one knows 12 – 8 = 4); and creating equivalent but easier or known sums (e.g., adding 6 + 7 by creating the known equivalent 6 + 6 + 1 = 12 + 1 = 13).	4-6, 4-7, 6-1, 6-2, 6-3, 6-4, 6-5, 7-1, 7-2, 7-3, 7-4, 16-1, 16-2, 16-3, 16-5, 16-6, 17-1, 17-2, 17-3, 17-4
Work with addition and subtraction equations.		
1.OA.7.	Understand the meaning of the equal sign, and determine if equations involving addition and subtraction are true or false. For example, which of the following equations are true and which are false? 6 = 6, 7 = 8 – 1, 5 + 2 = 2 + 5, 4 + 1 = 5 + 2.	3-4, 4-4, 6-1, 11-4, CC-1, CC-3
1.OA.8	Determine the unknown whole number in an addition or subtraction equation relating to three whole numbers. For example, determine the unknown number that makes the equation true in each of the equations 8 + ? = 11, 5 = � – 3, 6 + 6 = �.	3-4, 4-1, 4-2, 4-3, 4-5, 4-6, 4-7, 5-4, 6-2, 6-3, 6-4, 6-5, 7-2, 7-3, 7-4, 16-3, 16-5, 16-6, 17-2, 17-3, 17-4, CC-3

Standards for Mathematical Content Grade 1		Where to find in *enVisionMATH* ©2009/2011
Number and Operations in Base Ten		
Extend the counting sequence.		
1.NBT.1	Count to 120, starting at any number less than 120. In this range, read and write numerals and represent a number of objects with a written numeral.	1-1, 1-2, 1-3, 1-4, 1-5, 1-6, 10-3, 10-4, 10-5, 11-1, 11-2, 11-3, 11-4
Understand place value.		
1.NBT.2	Understand that the two digits of a two-digit number represent amounts of tens and ones. Understand the following as special cases:	1-3, 10-1, 11-1, 11-2, 11-3, 11-4, 11-5, 11-6, 12-2
1.NBT.2.a	10 can be thought of as a bundle of ten ones — called a “ten.”	11-1, 11-2, 11-3, 11-5, 11-6
1.NBT.2.b	The numbers from 11 to 19 are composed of a ten and one, two, three, four, five, six, seven, eight, or nine ones.	1-3, 11-1, 11-3
1.NBT.2.c	The numbers 10, 20, 30, 40, 50, 60, 70, 80, 90 refer to one, two, three, four, five, six, seven, eight, or nine tens (and 0 ones).	10-3, 11-2, 11-3
1.NBT.3	Compare two two-digit numbers based on meanings of the tens and ones digits, recording the results of comparisons with the symbols >, =, and <.	2-1, 12-3, 12-4, 12-5, 12-6, 12-7, 12-8
Use place value understanding and properties of operations to add and subtract.		
1.NBT.4	Add within 100, including adding a two-digit number and a one-digit number, and adding a two-digit number and a multiple of 10, using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; relate the strategy to a written method and explain the reasoning used. Understand that in adding two-digit numbers, one adds tens and tens, ones and ones; and sometimes it is necessary to compose a ten.	12-1, 12-2, 20-1, 20-2, 20-3, 20-4, CC-10
1.NBT.5	Given a two-digit number, mentally find 10 more or 10 less than the number, without having to count; explain the reasoning used.	12-1, 20-5, 20-6
1.NBT.6	Subtract multiples of 10 in the range 10-90 from multiples of 10 in the range 10-90 (positive or zero differences), using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; relate the strategy to a written method and explain the reasoning used.	12-1, 20-1, 20-2, 20-3, 20-4, 20-5, 20-6, 20-7, CC-11, CC-12

Standards for Mathematical Content Grade 1		Where to find in *enVisionMATH* ©2009/2011
Measurement and Data		
Measure lengths indirectly and by iterating length units.		
1.MD.1	Order three objects by length; compare the lengths of two objects indirectly by using a third object.	14-1, CC-6
1.MD.2	Express the length of an object as a whole number of length units, by laying multiple copies of a shorter object (the length unit) end to end; understand that the length measurement of an object is the number of same-size length units that span it with no gaps or overlaps. Limit to contexts where the object being measured is spanned by a whole number of length units with no gaps or overlaps.	14-2, 14-3, 14-4, 14-5, CC-7
Tell and write time.		
1.MD.3	Tell and write time in hours and half-hours using analog and digital clocks.	15-1, 15-2, 15-3
Represent and interpret data.		
1.MD.4	Organize, represent, and interpret data with up to three categories; ask and answer questions about the total number of data points, how many in each category, and how many more or less are in one category than in another.	18-1, 18-2, 18-3, 18-5, 18-6, 18-7, 18-8

Standards for Mathematical Content Grade 1		Where to find in *enVisionMATH* ©2009/2011
Geometry		
Reason with shapes and their attributes.		
1.G.1	Distinguish between defining attributes (e.g., triangles are closed and three-sided) versus non-defining attributes (e.g., color, orientation, overall size); build and draw shapes to possess defining attributes.	8-1, 8-2, 8-9, 8-10, 8-11
1.G.2	Compose two-dimensional shapes (rectangles, squares, trapezoids, triangles, half-circles, and quarter-circles) or three-dimensional shapes (cubes, right rectangular prisms, right circular cones, and right circular cylinders) to create a composite shape, and compose new shapes from the composite shape. (Students do not need to learn formal names such as "right rectangular prism.")	8-3, 8-4, CC-4, CC-5
1.G.3	Partition circles and rectangles into two and four equal shares, describe the shares using the words halves, fourths, and quarters, and use the phrases half of, fourth of, and quarter of. Describe the whole as two of, or four of the shares. Understand for these examples that decomposing into more equal shares creates smaller shares.	19-1, 19-2, 19-3, CC-9

Common Core State Standards
Standards for Mathematical Practice

The Standards for Mathematical Practice are an important part of the Common Core State Standards. They describe varieties of proficiency that teachers should focus on developing in their students. These practices draw from the NCTM process standards of problem solving, reasoning and proof, communication, representation, and connections and the strands of mathematical proficiency specified in the National Research Council's report *Adding It Up*: adaptive reasoning, strategic competence, conceptual understanding, procedural fluency, and productive disposition.

For each of the Standards for Mathematical Practice presented in the text that follows, is a explanation of the different features and elements of Pearson's *Scott Foresman • Addison Wesley enVisionMATH*® that help students develop mathematical proficiency.

1. Make sense of problems and persevere in solving them.

Mathematically proficient students start by explaining to themselves the meaning of a problem and looking for entry points to its solution. They analyze givens, constraints, relationships, and goals. They make conjectures about the form and meaning of the solution and plan a solution pathway rather than simply jumping into a solution attempt. They consider analogous problems, and try special cases and simpler forms of the original problem in order to gain insight into its solution. They monitor and evaluate their progress and change course if necessary. Older students might, depending on the context of the problem, transform algebraic expressions or change the viewing window on their graphing calculator to get the information they need. Mathematically proficient students can explain correspondences between equations, verbal descriptions, tables, and graphs or draw diagrams of important features and relationships, graph data, and search for regularity or trends. Younger students might rely on using concrete objects or pictures to help conceptualize and solve a problem. Mathematically proficient students check their answers to problems using a different method, and they continually ask themselves, "Does this make sense? "They can understand the approaches of others to solving complex problems and identify correspondences between different approaches.

Scott Foresman • Addison Wesley enVisionMATH is built on a foundation of problem-based instruction that has sense-making at its heart. The structure of each lesson facilitates students' implementation of this process. Every lesson begins with *Problem-Based Interactive Learning,* an activity in which students are presented a problem to solve. They interact with their peers and teachers to make sense of the problem presented and to look for a workable solution. A second feature of each lesson are the Problem Solving Exercises for which students persevere to find solutions for each exercise. In each topic is at least one Problem Solving lesson with a primary focus of honing students' sense-making and problem-solving skills.

Throughout the program; for examples, see *enVisionMATH* Grade 2 Lessons 1-7, 2-7, 3-6, 4-8, 6-6, 7-6, 8-9, 9-9, 12-8, 16-7

2 Reason abstractly and quantitatively.

Mathematically proficient students make sense of quantities and their relationships in problem situations. They bring two complementary abilities to bear on problems involving quantitative relationships: the ability to decontextualize—to abstract a given situation and represent it symbolically and manipulate the representing symbols as if they have a life of their own, without necessarily attending to their referents—and the ability to contextualize, to pause as needed during the manipulation process in order to probe into the referents for the symbols involved. Quantitative reasoning entails habits of creating a coherent representation of the problem at hand; considering the units involved; attending to the meaning of quantities, not just how to compute them; and knowing and flexibly using different properties of operations and objects.

Reasoning is another important theme of the *Scott Foresman • Addison Wesley enVisionMATH* Program. In most lessons, the Visual Learning Bridge presents a situation and students are shown how the situation can be represented numerically or algebraically. Later in a lesson, students have opportunities to work on their own to represent situations symbolically. Through the solving process, students are reminded to check back to the problem situation with the Reasonableness exercises. In the **Do You Understand** part of the Guided Practice, students are often asked to consider the meaning of different parts of an expression or equation. The *Journal* Exercises also help students to be thinking and reasoning abstractly and quantitatively.

Throughout the program; for examples, see *enVisionMATH* Grade 2 Lessons 1-5, 3-4, 5-1, 6-2, 6-6, 8-8, 10-2, 10-10, 11-5, 13-2, 13-4

3 Construct viable arguments and critique the reasoning of others.

Mathematically proficient students understand and use stated assumptions, definitions, and previously established results in constructing arguments. They make conjectures and build a logical progression of statements to explore the truth of their conjectures. They are able to analyze situations by breaking them into cases, and can recognize and use counterexamples. They justify their conclusions, communicate them to others, and respond to the arguments of others. They reason inductively about data, making plausible arguments that take into account the context from which the data arose. Mathematically proficient students are also able to compare the effectiveness of two plausible arguments, distinguish correct logic or reasoning from that which is flawed, and—if there is a flaw in an argument—explain what it is. Elementary students can construct arguments using concrete referents such as objects, drawings, diagrams, and actions. Such arguments can make sense and be correct, even though they are not generalized or made formal until later grades. Later, students learn to determine domains to which an argument applies. Students at all grades can listen or read the arguments of others, decide whether they make sense, and ask useful questions to clarify or improve the arguments.

Consistent with a focus on reasoning and sense-making is a focus on critical reasoning – argumentation and critique of arguments. In Pearson's *Scott Foresman • Addison Wesley enVisionMATH,* the Problem-Based Interactive Learning affords students opportunities to share with classmates their thinking about problems, their solutions, and their reasoning about the solutions. The *Journal* Exercises help students develop foundational critical reasoning skills by having them construct explanations for processes. Articulating clearly an explanation for a process is a stepping stone to critical analysis and reasoning of both their own processes and those of others.

Throughout the program; for examples, see *enVisionMATH* Grade 2 Lessons 2-5, 3-1, 6-6, 10-7, 11-4, 11-10

4 Model with mathematics.

Mathematically proficient students can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace. In early grades, this might be as simple as writing an addition equation to describe a situation. In middle grades, a student might apply proportional reasoning to plan a school event or analyze a problem in the community. By high school, a student might use geometry to solve a design problem or use a function to describe how one quantity of interest depends on another. Mathematically proficient students who can apply what they know are comfortable making assumptions and approximations to simplify a complicated situation, realizing that these may need revision later. They are able to identify important quantities in a practical situation and map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts and formulas. They can analyze those relationships mathematically to draw conclusions. They routinely interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose.

Students in Pearson's *Scott Foresman • Addison Wesley enVisionMATH,* are introduced to mathematical modeling in the early grades. They first use manipulatives and drawings and then equations to model addition and subtraction situations. The Visual Learning Bridge and Visual Learning Animation often present real-world situations and students are shown how these can be modeled mathematically. In later years, students expand their modeling skills to include other graphical representations such as tables, graphs, as well as equations.

Throughout the program; for examples, see *enVisionMATH* Grade 2 Lessons 1-1, 2-8, 3-3, 4-2, 4-4, 8-2, 8-9, 10-2, 12-5

5 Use appropriate tools strategically.

Mathematically proficient students consider the available tools when solving a mathematical problem. These tools might include pencil and paper, concrete models, a ruler, a protractor, a calculator, a spreadsheet, a computer algebra system, a statistical package, or dynamic geometry software. Proficient students are sufficiently familiar with tools appropriate for their grade or course to make sound decisions about when each of these tools might be helpful, recognizing both the insight to be gained and their limitations. For example, mathematically proficient high school students analyze graphs of functions and solutions generated using a graphing calculator. They detect possible errors by strategically using estimation and other mathematical knowledge. When making mathematical models, they know that technology can enable them to visualize the results of varying assumptions, explore consequences, and compare predictions with data. Mathematically proficient students at various grade levels are able to identify relevant external mathematical resources, such as digital content located on a website, and use them to pose or solve problems. They are able to use technological tools to explore and deepen their understanding of concepts.

Students become fluent in the use of a wide assortment of tools ranging from physical objects, including manipulatives, rulers, protractors, and even pencil and paper, to technological tools, such as etools, calculators and computers. As students become more familiar with the tools available to them, they are able to begin making decisions about which tools are more appropriate to solve different kinds of problems.

Throughout the program; for examples, see *enVisionMATH* Grade 2 Lessons 1-7, 4-4, 5-8, 6-4, 8-9, 11-2, 12-8, 13-6, 16-7

6 Attend to precision.

Mathematically proficient students try to communicate precisely to others. They try to use clear definitions in discussion with others and in their own reasoning. They state the meaning of the symbols they choose, including using the equal sign consistently and appropriately. They are careful about specifying units of measure, and labeling axes to clarify the correspondence with quantities in a problem. They calculate accurately and efficiently, express numerical answers with a degree of precision appropriate for the problem context. In the elementary grades, students give carefully formulated explanations to each other. By the time they reach high school they have learned to examine claims and make explicit use of definitions.

Students are expected to use mathematical terms and symbols with precision. Key terms and concepts are highlighted in each lesson. In the **Do You Understand** feature, students often revisit these key terms and provide explicit definitions or explanations of the terms. For the *Journal* Exercises, students are to provide clear explanations of terms, concepts, or processes and to use new terms accurately and precisely.

Throughout the program; for examples, see *enVisionMATH* Grade 2 Lessons 5-2, 10-3, 13-5, 16-2, 16-4

7 Look for and make use of structure.

Mathematically proficient students look closely to discern a pattern or structure .Young students, for example, might notice that three and seven more is the same amount as seven and three more, or they may sort a collection of shapes according to how many sides the shapes have. Later, students will see 7 × 8 equals the well remembered 7 × 5 × 7 × 3, in preparation for learning about the distributive property. In the expression $x^2 + 9x + 14$, older students can see the 14 as 2 × 7 and the 9 as 2 + 7.They recognize the significance of an existing line in a geometric figure and can use the strategy of drawing an auxiliary line for solving problems. They also can step back for an overview and shift perspective. They can see complicated things, such as some algebraic expressions, as single objects or as being composed of several objects. For example, they can see $5 \;\; 3(x \;\; y)^2$ as 5 minus a positive number times a square and use that to realize that its value cannot be more than 5 for any real numbers x and y.

Throughout the program, students are encouraged to look for patterns and structure as they look to develop solution plans. In the *Look for a Pattern* Problem-Solving lessons, children in the early years develop a sense of patterning with visual and physical objects.

As students mature in their mathematical thinking, they look for patterns in numerical operations by focusing on place value and properties of operations. From this focus on looking for and recognizing patterns, students become well-equip to draw from these patterns to formalize their thinking about the structure of operations.

Throughout the program; for examples, see *enVisionMATH* Grade 2 Lessons 1-6, 2-2, 3-4, 6-1, 7-1, 12-2

8 Look for and express regularity in repeated reasoning.

Mathematically proficient students notice if calculations are repeated, and look both for general methods and for shortcuts. Upper elementary students might notice when dividing 25 by 11 that they are repeating the same calculations over and over again, and conclude they have a repeating decimal. By paying attention to the calculation of slope as they repeatedly check whether points are on the line through (1, 2) with slope 3, middle school students might abstract the equation $(y - 2)/(x - 1) = 3$. Noticing the regularity in the way terms cancel when expanding $(x - 1)(x + 1)$, $(x - 1)(x^2 + x + 1)$, and $(x - 1)(x^3 + x^2 + x + 1)$ might lead them to the general formula for the sum of a geometric series. As they work to solve a problem, mathematically proficient students maintain oversight of the process, while attending to the details. They continually evaluate the reasonableness of their intermediate results.

Once again, throughout the program as a whole, students are prompted to look for repetition in computations to look for shortcuts that can make the problem-solving process more efficient. Students are prompted to think about problems they encountered previously that may share features or processes. They are encouraged to draw on the solution plan developed for that problem, and as their mathematical thinking matures, to look for generalizations that can be applied to other problem situations. The **Problem-Based Interactive Learning** activities offer students opportunities to look for regularity in the way operations behave.

Throughout the program; for examples, see *enVisionMATH* Grade 2 Lessons 5-7, 8-7, 8-8, 9-8, 10-9, 15-9

Correlation of Standards for Mathematical Content

enVisionMATH™ Grade 2

The following shows the alignment of *enVisionMATH* Grade 2 ©2009/2011 to the Common Core State Standards for Grade 2. Included in this correlation are the supplemental lessons that will be available as part of the transitional support that Pearson is providing. These lessons will be available in the summer 2011.

Standards for Mathematical Content Grade 2		Where to find in enVisionMATH ©2009/2011
Operations and Algebraic Thinking		
Represent and solve problems involving addition and subtraction.		
2.OA.1	Use addition and subtraction within 100 to solve one- and two-step word problems involving situations of adding to, taking from, putting together, taking apart, and comparing, with unknowns in all positions, e.g., by using drawings and equations with a symbol for the unknown number to represent the problem.	1-1, 1-2, 1-3, 1-4, 1-5, 1-6, 1-7, 2-1, 2-2, 2-3, 2-4, 2-5, 2-8, 3-1, 3-2, 3-3, 3-4, 3-6, 6-1, 6-2, 6-3, 6-4, 7-3, 7-4, 7-5, 8-1, 8-7, 9-8, 10-7, 15-6
Add and subtract within 20.		
2.OA.2	Fluently add and subtract within 20 using mental strategies. By end of Grade 2, know from memory all sums of two one-digit numbers. *(See standard 1.OA.6 for a list of mental strategies.)*	2-1, 2-2, 2-3, 3-1, 3-2, 3-3, 3-4
Work with equal groups of objects to gain foundations for multiplication.		
2.OA.3	Determine whether a group of objects (up to 20) has an odd or even number of members, e.g., by pairing objects or counting them by 2s; write an equation to express an even number as a sum of two equal addends.	4-9
2.OA.4	Use addition to find the total number of objects arranged in rectangular arrays with up to 5 rows and up to 5 columns; write an equation to express the total as a sum of equal addends.	19-2, 19-3, 19-5, 19-6

Standards for Mathematical Content Grade 2		Where to find in enVisionMATH ©2009/2011
Number and Operations in Base Ten		
Understand place value.		
2.NBT.1	Understand that the three digits of a three-digit number represent amounts of hundreds, tens, and ones; e.g., 706 equals 7 hundreds, 0 tens, and 6 ones. Understand the following as special cases:	4-2, 4-3, 17-1, 17-2, 17-3
2.NBT.1.a	100 can be thought of as a bundle of ten tens — called a "hundred."	17-1, 17-3
2.NBT.1.b	The numbers 100, 200, 300, 400, 500, 600, 700, 800, 900 refer to one, two, three, four, five, six, seven, eight, or nine hundreds (and 0 tens and 0 ones).	17-1, 17-2, 17-3
2.NBT.2	Count within 1000; skip-count by 5s, 10s, and 100s.	4-8, 17-1, 17-5, 17-6a
2.NBT.3	Read and write numbers to 1000 using base-ten numerals, number names, and expanded form.	4-2, 4-3, 17-2, 17-3
2.NBT.4	Compare two three-digit numbers based on meanings of the hundreds, tens, and ones digits, using >, =, and < symbols to record the results of comparisons.	17-6, 17-8
Use place value understanding and properties of operations to add and subtract.		
2.NBT.5	Fluently add and subtract within 100 using strategies based on place value, properties of operations, and/or the relationship between addition and subtraction.	1-6, 2-1, 2-2, 2-3, 2-4, 2-5, 2-6, 2-7, 3-1, 3-2, 3-3, 3-4, 3-5, 5-6, 5-8, 6-1, 6-2, 6-3, 6-4, 6-5a, 6-5, 7-1, 7-2, 7-3a,7-3, 7-4, 8-1, 8-2, 8-3, 8-4, 8-5, 8-6, 9-1, 9-2, 9-3, 9-4, 9-5, 9-6a, 9-6, 9-7, 10-1, 10-3, 10-4, 10-6
2.NBT.6	Add up to four two-digit numbers using strategies based on place value and properties of operations.	8-1, 8-2, 8-3, 8-4, 8-5, 8-6a, 8-6, 9-6a
2.NBT.7	Add and subtract within 1000, using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; relate the strategy to a written method. Understand that in adding or subtracting three-digit numbers, one adds or subtracts hundreds and hundreds, tens and tens, ones and ones; and sometimes it is necessary to compose or decompose tens or hundreds.	18-1a, 18-1, 18-2, 18-3, 18-4, 18-5a, 18-5, 18-6, 18-7, 18-8
2.NBT.8	Mentally add 10 or 100 to a given number 100–900, and mentally subtract 10 or 100 from a given number 100–900.	6-5a, 7-3a,17-5, 17-5, 18-1a, 18-1, 18-5a, 18-5

Standards for Mathematical Content Grade 2		Where to find in enVisionMATH ©2009/2011
2.NBT.9	Explain why addition and subtraction strategies work, using place value and the properties of operations. *(Explanations may be supported by drawings or objects.)*	2-1, 2-2, 2-3, 2-4, 2-5, 2-6, 2-7, 3-1, 3-2, 3-3, 3-4, 3-5, 6-1, 6-2, 6-3, 6-4, 6-5a, 7-1, 7-2, 7-3a, 7-3, 7-4, 8-1, 8-2, 8-3, 8-4, 8-5, 8-6a, 8-6, 9-1, 9-2, 9-3, 9-4, 9-5, 9-6a, 9-6, 9-7, 10-1, 10-3, 10-4, 10-6, 18-1, 18-3, 18-4, 18-5, 18-7, 18-8

Standards for Mathematical Content Grade 2		Where to find in enVisionMATH ©2009/2011
Measurement and Data		
Measure and estimate lengths in standard units.		
2.MD.1	Measure the length of an object by selecting and using appropriate tools such as rulers, yardsticks, meter sticks, and measuring tapes.	13-4a, 13-4, 13-5a, 13-5
2.MD.2	Measure the length of an object twice, using length units of different lengths for the two measurements; describe how the two measurements relate to the size of the unit chosen.	13-3, 13-6a
2.MD.3	Estimate lengths using units of inches, feet, centimeters, and meters.	13-4a, 13-4, 13-5a, 13-5
2.MD.4	Measure to determine how much longer one object is than another, expressing the length difference in terms of a standard length unit.	13-6c
Relate addition and subtraction to length.		
2.MD.5	Use addition and subtraction within 100 to solve word problems involving lengths that are given in the same units, e.g., by using drawings (such as drawings of rulers) and equations with a symbol for the unknown number to represent the problem.	13-6b
2.MD.6	Represent whole numbers as lengths from 0 on a number line diagram with equally spaced points corresponding to the numbers 0, 1, 2,..., and represent whole-number sums and differences within 100 on a number line diagram.	8-6a, 9-6a
Work with time and money.		
2.MD.7	Tell and write time from analog and digital clocks to the nearest five minutes, using a.m. and p.m.	15-1, 15-2
2.MD.8	Solve word problems involving dollar bills, quarters, dimes, nickels, and pennies, using $ and ¢ symbols appropriately. *Example: If you have 2 dimes and 3 pennies, how many cents do you have?*	5-1, 5-2, 5-3, 5-4, 5-5, 5-6, 10-7
Represent and interpret data.		
2.MD.9	Generate measurement data by measuring lengths of several objects to the nearest whole unit, or by making repeated measurements of the same object. Show the measurements by making a line plot, where the horizontal scale is marked off in whole-number units.	13-4, 13-5, 13-6d
2.MD.10	Draw a picture graph and a bar graph (with single-unit scale) to represent a data set with up to four categories. Solve simple put-together, take-apart, and compare problems using information presented in a bar graph.	16-1, 16-2, 16-3, 16-7, 18-9

Standards for Mathematical Content Grade 2		Where to find in enVisionMATH ©2009/2011
Geometry		
Reason with shapes and their attributes.		
2.G.1	Recognize and draw shapes having specified attributes, such as a given number of angles or a given number of equal faces. Identify triangles, quadrilaterals, pentagons, hexagons, and cubes. *(Sizes are compared directly or visually, not compared by measuring.)*	11-1, 11-2, 11-3a, 11-3, 11-4, 11-8,
2.G.2	Partition a rectangle into rows and columns of same-size squares and count to find the total number of them.	11-5a, 13-7, 13-8, 19-5
2.G.3	Partition circles and rectangles into two, three, or four equal shares, describe the shares using the words *halves*, *thirds*, *half of*, *a third of*, etc., and describe the whole as two halves, three thirds, four fourths. Recognize that equal shares of identical wholes need not have the same shape.	12-1, 12-2, 12-3

Common Core State Standards
Standards for Mathematical Practice
Grade 3

The Standards for Mathematical Practice are an important part of the Common Core State Standards. They describe varieties of proficiency that teachers should focus on developing in their students. These practices draw from the NCTM process standards of problem solving, reasoning and proof, communication, representation, and connections and the strands of mathematical proficiency specified in the National Research Council's report *Adding It Up*: adaptive reasoning, strategic competence, conceptual understanding, procedural fluency, and productive disposition.

For each of the Standards for Mathematical Practice presented in the text that follows is a explanation of the different features and elements of Pearson's *Scott Foresman • Addison Wesley enVisionMATH*TM that help students develop mathematical proficiency.

1. Make sense of problems and persevere in solving them.

Mathematically proficient students start by explaining to themselves the meaning of a problem and looking for entry points to its solution. They analyze givens, constraints, relationships, and goals. They make conjectures about the form and meaning of the solution and plan a solution pathway rather than simply jumping into a solution attempt. They consider analogous problems, and try special cases and simpler forms of the original problem in order to gain insight into its solution. They monitor and evaluate their progress and change course if necessary. Older students might, depending on the context of the problem, transform algebraic expressions or change the viewing window on their graphing calculator to get the information they need. Mathematically proficient students can explain correspondences between equations, verbal descriptions, tables, and graphs or draw diagrams of important features and relationships, graph data, and search for regularity or trends. Younger students might rely on using concrete objects or pictures to help conceptualize and solve a problem. Mathematically proficient students check their answers to problems using a different method, and they continually ask themselves, "Does this make sense? "They can understand the approaches of others to solving complex problems and identify correspondences between different approaches.

Scott Foresman • Addison Wesley enVisionMATH, is built on a foundation of problem-based instruction that has sense-making at its heart. The Problem Solving Handbook, found on pages xviii-xxix, presents to students a 3-phase process that begins with making sense of the problem to solve. The first phase of the process has students ask themselves, *What am I trying to find? What do I know?* to help them identify the givens and constraints of a problem situation. In the second phase, the plan and solve phase, students decide on a solution plan. The Problem-Solving Recording Sheet, a reproducible teaching resource, provides students with a useful structured outline to help them become fluent with thinking about (making sense of) a problem and planning a workable solution pathway.

The structure of each lesson facilitates students' implementation of this process. Every lesson begins with **Problem-Based Interactive Learning,** an activity in which students are presented a problem to solve. They interact with their peers and teachers to make sense of the problem presented and to look for a workable solution. A second feature of each lesson are the Problem Solving exercises for which students persevere to find solutions for each exercise. In each topic is at least one Problem Solving lesson with a primary focus of honing students' sense-making and problem-solving skills.

Throughout the program; for examples, see *enVisionMATH* Grade 3 Lessons 3-5, 3-10, 4-5, 5-7, 6-7, 6-9, 8-9, 14-6, 16-6.

2 Reason abstractly and quantitatively.

Mathematically proficient students make sense of quantities and their relationships in problem situations. They bring two complementary abilities to bear on problems involving quantitative relationships: the ability to decontextualize—to abstract a given situation and represent it symbolically and manipulate the representing symbols as if they have a life of their own, without necessarily attending to their referents—and the ability to contextualize, to pause as needed during the manipulation process in order to probe into the referents for the symbols involved. Quantitative reasoning entails habits of creating a coherent representation of the problem at hand; considering the units involved; attending to the meaning of quantities, not just how to compute them; and knowing and flexibly using different properties of operations and objects.

Scott Foresman • Addison Wesley enVisionMATH Program provides scaffolded instruction to help students develop both quantitative and abstract reasoning. In the Visual Learning Bridge students learn how to represent the given situation numerically or algebraically. Later in a lesson, students have opportunities to reason abstractly as they endeavor to represent situations symbolically. Throughout the solving process, students are reminded to check back to the problem situation with the **Reasonableness** exercises. In the **Do You Understand** part of the Guided Practice, students gain experiences with quantitative reasoning as they consider the meaning of different parts of an expression or equation. Throughout the exercise sets are Reasoning exercises that focus students' attention on the structure or meaning of an operation rather than the solution.

Throughout the program; for examples, see *enVisionMATH* Grade 3 Lessons 6-9, 7-2, 16-3

3 Construct viable arguments and critique the reasoning of others.

Mathematically proficient students understand and use stated assumptions, definitions, and previously established results in constructing arguments. They make conjectures and build a logical progression of statements to explore the truth of their conjectures. They are able to analyze situations by breaking them into cases, and can recognize and use counterexamples. They justify their conclusions, communicate them to others, and respond to the arguments of others. They reason inductively about data, making plausible arguments that take into account the context from which the data arose. Mathematically proficient students are also able to compare the effectiveness of two plausible arguments, distinguish correct logic or reasoning from that which is flawed, and—if there is a flaw in an argument—explain what it is. Elementary students can construct arguments using concrete referents such as objects, drawings, diagrams, and actions. Such arguments can make sense and be correct, even though they are not generalized or made formal until later grades. Later, students learn to determine domains to which an argument applies. Students at all grades can listen or read the arguments of others, decide whether they make sense, and ask useful questions to clarify or improve the arguments.

Consistent with a focus on reasoning and sense-making is a focus on critical reasoning – argumentation and critique of arguments. In Pearson's *Scott Foresman • Addison Wesley enVisionMATH,* the Problem-Based Interactive Learning affords students opportunities to share with classmates their thinking about problems, their solutions, and their reasoning about the solutions. The many Reasoning exercises found throughout the program specifically call for students to justify or explain their solutions. The **Writing to Explain** exercises help students develop foundational critical reasoning skills by having them construct explanations for processes. Articulating clearly an explanation for a process is a stepping stone to critical analysis and reasoning of both their own processes and those of others.

Throughout the program; for examples, see *enVisionMATH* Grade 3 Lessons 2-6, 2-9, 3-9, 7-5, 8-3, 10-5, 11-1, 14-7

4 Model with mathematics.

Mathematically proficient students can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace. In early grades, this might be as simple as writing an addition equation to describe a situation. In middle grades, a student might apply proportional reasoning to plan a school event or analyze a problem in the community. By high school, a student might use geometry to solve a design problem or use a function to describe how one quantity of interest depends on another. Mathematically proficient students who can apply what they know are comfortable making assumptions and approximations to simplify a complicated situation, realizing that these may need revision later. They are able to identify important quantities in a practical situation and map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts and formulas. They can analyze those relationships mathematically to draw conclusions. They routinely interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose.

Students in Pearson's *Scott Foresman • Addison Wesley enVisionMATH,* are introduced to mathematical modeling in the early grades. They first use manipulatives and drawings and then equations to model addition and subtraction situations. The Visual Learning Bridge and Visual Learning Animation often present real-world situations and students are shown how these can be modeled mathematically. In later years, students expand their modeling skills to include other graphical representations such as tables, graphs, as well as equations.

Throughout the program; for examples, see *enVisionMATH* Grade 3 Lessons 2-1, 3-5, 6-7, 8-9, 11-4, 13-1

5 Use appropriate tools strategically.

Mathematically proficient students consider the available tools when solving a mathematical problem. These tools might include pencil and paper, concrete models, a ruler, a protractor, a calculator, a spreadsheet, a computer algebra system, a statistical package, or dynamic geometry software. Proficient students are sufficiently familiar with tools appropriate for their grade or course to make sound decisions about when each of these tools might be helpful, recognizing both the insight to be gained and their limitations. For example, mathematically proficient high school students analyze graphs of functions and solutions generated using a graphing calculator. They detect possible errors by strategically using estimation and other mathematical knowledge. When making mathematical models, they know that technology can enable them to visualize the results of varying assumptions, explore consequences, and compare predictions with data. Mathematically proficient students at various grade levels are able to identify relevant external mathematical resources, such as digital content located on a website, and use them to pose or solve problems. They are able to use technological tools to explore and deepen their understanding of concepts.

Students become fluent in the use of a wide assortment of tools ranging from physical objects, including manipulatives, rulers, protractors, and even pencil and paper, to technological tools, such as etools, calculators and computers. As students become more familiar with the tools available to them, they are able to begin making decisions about which tools are more appropriate to solve different kinds of problems.

Throughout the program; for examples, see *enVisionMATH* Grade 3 Lessons 3-2, 3-7, 5-1, 10-5, 11-3

6 Attend to precision.

Mathematically proficient students try to communicate precisely to others. They try to use clear definitions in discussion with others and in their own reasoning. They state the meaning of the symbols they choose, including using the equal sign consistently and appropriately. They are careful about specifying units of measure, and labeling axes to clarify the correspondence with quantities in a problem. They calculate accurately and efficiently, express numerical answers with a degree of precision appropriate for the problem context. In the elementary grades, students give carefully formulated explanations to each other. By the time they reach high school they have learned to examine claims and make explicit use of definitions.

Students are expected to use mathematical terms and symbols with precision. Key terms and concepts are highlighted in each lesson. In the **Do You Understand** feature, students often revisit these key terms and provide explicit definitions or explanations of the terms. For the Writing to Explain and Think About a Process exercises, students are to provide clear explanations of terms, concepts, or processes and to use new terms accurately and precisely. Students are reminded to use appropriate units of measure when working through solutions and accurate labels on axes when making graphs to represent solutions.

Throughout the program; for examples, see *enVisionMATH* Grade 3 Lessons 1-5, 3-9, 4-5, 8-3, 12-2, 16-3

7 Look for and make use of structure.

Mathematically proficient students look closely to discern a pattern or structure. Young students, for example, might notice that three and seven more is the same amount as seven and three more, or they may sort a collection of shapes according to how many sides the shapes have. Later, students will see 7 × 8 equals the well remembered 7 × 5 × 7 × 3, in preparation for learning about the distributive property. In the expression $x^2 + 9x + 14$, older students can see the 14 as 2 × 7 and the 9 as 2 + 7.They recognize the significance of an existing line in a geometric figure and can use the strategy of drawing an auxiliary line for solving problems. They also can step back for an overview and shift perspective. They can see complicated things, such as some algebraic expressions, as single objects or as being composed of several objects. For example, they can see 5 3(x y)^2 as 5 minus a positive number times a square and use that to realize that its value cannot be more than 5 for any real numbers x and y.

Throughout the program, students are encouraged to look for structure as they develop solution plans. In the **Look for a Pattern Problem-Solving** lessons, children in the early years develop a sense of patterning with visual and physical objects.

As students mature in their mathematical thinking, they look for structure in numerical operations by focusing on place value and properties of operations. From this focus on looking for and recognizing structure, students become well-equip to draw from patterns to formalize their thinking about the structure of operations.

Throughout the program; for examples, see *enVisionMATH* Grade 3 Lessons 5-1, 8-4, 16-6, 14-6

8 Look for and express regularity in repeated reasoning.

Mathematically proficient students notice if calculations are repeated, and look both for general methods and for shortcuts. Upper elementary students might notice when dividing 25 by 11 that they are repeating the same calculations over and over again, and conclude they have a repeating decimal. By paying attention to the calculation of slope as they repeatedly check whether points are on the line through (1, 2) with slope 3, middle school students might abstract the equation $(y – 2)/(x – 1) = 3$. Noticing the regularity in the way terms cancel when expanding $(x – 1)(x + 1)$, $(x – 1)(x^2 + x + 1)$, and $(x – 1)(x^3 + x^2 + x + 1)$ might lead them to the general formula for the sum of a geometric series. As they work to solve a problem, mathematically proficient students maintain oversight of the process, while attending to the details. They continually evaluate the reasonableness of their intermediate results.

Once again, throughout the program as a whole, students are prompted to look for repetition in computations to derive shortcuts that can make the problem-solving process more efficient. Students are prompted to think about problems they encountered previously that may share features or processes. They are encouraged to draw on the solution plan developed for that problem, and as their mathematical thinking matures, to look for generalizations that can be applied to other problem situations. The **Problem-Based Interactive Learning** activities offer students opportunities to look for regularity in the way operations behave.

Throughout the program; for examples, see *enVisionMATH* Grade 3 Lessons 2-4, 3-4, 3-8, 5-2

Correlation of Standards for Mathematical Content

enVisionMATH™ Grade 3

The following shows the alignment of *enVisionMATH* Grade 3 ©2009/2011 to the Common Core State Standards for Grade 3. Included in this correlation are the supplemental lessons that will be available as part of the transitional support that Pearson is providing. These lessons will be available in the summer 2011.

Standards for Mathematical Content Grade 3		Where to find in enVisionMATH ©2009/2011
Operations and Algebraic Thinking		
Represent and solve problems involving multiplication and division.		
3.OA.1	Interpret products of whole numbers, e.g., interpret 5 × 7 as the total number of objects in 5 groups of 7 objects each.	5-1, 5-2, 5-3a , 5-3, 5-4, 5-5, 6-5,
3.OA.2	Interpret whole-number quotients of whole numbers, e.g., interpret 56 ÷ 8 as the number of objects in each share when 56 objects are partitioned equally into 8 shares, or as a number of shares when 56 objects are partitioned into equal shares of 8 objects each.	7-1, 7-2, 7-3, 8-2
3.OA.3	Use multiplication and division within 100 to solve word problems in situations involving equal groups, arrays, and measurement quantities, e.g., by using drawings and equations with a symbol for the unknown number to represent the problem.1	5-1, 5-2, 5-3a, 5-3, 5-4, 5-5, 5-6, 5-7, 5-8, 5-9, 5-10, 6-1a, 6-1, 6-2, 6-3, 6-4, 6-5, 6-6, 6-7a, 6-7, 7-1, 7-2, 7-3, 7-4, 7-5, 8-1, 8-2, 8-3, 8-4, 8-5a , 8-5, 8-6, 12-10,
3.OA.4	Determine the unknown whole number in a multiplication or division equation relating three whole numbers.	5-2, 7-1, 7-2a, 7-3, 7-4a, 7-4, 7-5, 8-1, 8-2, 8-5a 8-5, 9-1
Understand properties of multiplication and the relationship between multiplication and division.		
3.OA.5	Apply properties of operations as strategies to multiply and divide. Examples: If 6 × 4 = 24 is known, then 4 × 6 = 24 is also known. (Commutative property of multiplication.) 3 × 5 × 2 can be found by 3 × 5 = 15, then 15 × 2 = 30, or by 5 × 2 = 10, then 3 × 10 = 30. (Associative property of multiplication.) Knowing that 8 × 5 = 40 and 8 × 2 = 16, one can find 8 × 7 as 8 × (5 + 2) = (8 × 5) + (8 × 2) = 40 + 16 = 56. (Distributive property.) (Students need not use formal terms for these properties.)	5-1, 5-2, 5-3a 5-4, 5-5, 6-2, 6-6, 18-4
3.OA.6	Understand division as an unknown-factor problem.	7-2a, 7-4a, 7-5, 8-2,

Standards for Mathematical Content Grade 3		Where to find in enVisionMATH ©2009/2011
		8-3, 8-4
Multiply and divide within 100.		
3.OA.7	Fluently multiply and divide within 100, using strategies such as the relationship between multiplication and division (e.g., knowing that 8 × 5 = 40, one knows 40 ÷ 5 = 8) or properties of operations. By the end of Grade 3, know from memory all products of two one-digit numbers.	8-1, 8-2, 8-3, 8-4
Solve problems involving the four operations, and identify and explain patterns in arithmetic.		
3.OA.8	Solve two-step word problems using the four operations. Represent these problems using equations with a letter standing for the unknown quantity. Assess the reasonableness of answers using mental computation and estimation strategies including rounding. (This standard is limited to problems posed with whole numbers and having whole-number answers; students should know how to perform operations in the conventional order when there are no parentheses to specify a particular order (Order of Operations).)	2-8, 2-9, 2-10, 3-5, 4-3, 4-4, 4-6, 5-7, 5-10, 6-1, 6-2, 6-36-7, 19-6
3.OA.9	Identify arithmetic patterns (including patterns in the addition table or multiplication table), and explain them using properties of operations.	2-1, 2-2, 5-2, 5-5, 5-6, 5-7, 5-8, 5-9, 6-5

Standards for Mathematical Content Grade 3		Where to find in enVisionMATH ©2009/2011
Number and Operations in Base Ten		
Use place value understanding and properties of operations to perform multi-digit arithmetic[1].		
3.NBT.1	Use place value understanding to round whole numbers to the nearest 10 or 100.	1-5a, 1-5b, 2-4, 2-8, 4-6
3.NBT.2	Fluently add and subtract within 1000 using strategies and algorithms based on place value, properties of operations, and/or the relationship between addition and subtraction.	2-1, 2-6, 2-7a, 2-7, 2-8, 2-9, 2-10, 3-5, 4-1a, 4-1, 4-2, 4-3a, 4-3, 4-4, 4-5
3.NBT.3	Multiply one-digit whole numbers by multiples of 10 in the range 10–90 (e.g., 9×80, 5×60) using strategies based on place value and properties of operations.	5-7, 5-8a, 18-1,

[1] (A range of algorithms may be used.)

Standards for Mathematical Content Grade 3		Where to find in enVisionMATH ©2009/2011
Number and Operations–Fractions[2]		
Develop understanding of fractions as numbers.		
3.NF.1	Understand a fraction 1/b as the quantity formed by 1 part when a whole is partitioned into b equal parts; understand a fraction a/b as the quantity formed by a parts of size 1/b.	12-1, 12-2a, 12-2, 12-3,
3.NF.2	Understand a fraction as a number on the number line; represent fractions on a number line diagram.	12-7
3.NF.2.a	Represent a fraction 1/b on a number line diagram by defining the interval from 0 to 1 as the whole and partitioning it into b equal parts. Recognize that each part has size 1/b and that the endpoint of the part based at 0 locates the number 1/b on the number line.	12-4, 12--7
3.NF.2.b	Represent a fraction a/b on a number line diagram by marking off a lengths 1/b from 0. Recognize that the resulting interval has size a/b and that its endpoint locates the number a/b on the number line.	12-7
3.NF.3	Explain equivalence of fractions in special cases, and compare fractions by reasoning about their size.	12-5, 12-6
3.NF.3.a	Understand two fractions as equivalent (equal) if they are the same size, or the same point on a number line.	12-5, 12-6, 12-7a, 12-7, 12-8a, 12-8b
3.NF.3.b	Recognize and generate simple equivalent fractions, e.g., 1/2 = 2/4, 4/6 = 2/3). Explain why the fractions are equivalent, e.g., by using a visual fraction model.	12-5, 12-6, 12-8b
3.NF.3.c	Express whole numbers as fractions, and recognize fractions that are equivalent to whole numbers. Examples: Express 3 in the form 3 = 3/1; recognize that 6/1 = 6; locate 4/4 and 1 at the same point of a number line diagram.	12-8b, 12-8c
3.NF.3.d	Compare two fractions with the same numerator or the same denominator by reasoning about their size. Recognize that comparisons are valid only when the two fractions refer to the same whole. Record the results of comparisons with the symbols >, =, or <, and justify the conclusions, e.g., by using a visual fraction model.	12-5a, 12-7a, 12-8a

[2] (Grade 3 expectations in this domain are limited to fractions with *denominators 2, 3, 4, 6, and 8.)*

Standards for Mathematical Content Grade 3		Where to find in enVisionMATH ©2009/2011
Measurement and Data		
Solve problems involving measurement and estimation of intervals of time, liquid volumes, and masses of objects.		
3.MD.1	Tell and write time to the nearest minute and measure time intervals in minutes. Solve word problems involving addition and subtraction of time intervals in minutes, e.g., by representing the problem on a number line diagram.	17-1, 17-2, 17-3, 17-4, 17-6
3.MD.2	Measure and estimate liquid volumes and masses of objects using standard units of grams (g), kilograms (kg), and liters (l). Add, subtract, multiply, or divide to solve one-step word problems involving masses or volumes that are given in the same units, e.g., by using drawings (such as a beaker with a measurement scale) to represent the problem. (Excludes compound units such as cm3 and finding the geometric volume of a container. Excludes multiplicative comparison problems (problems involving notions of "times as much").)	14-4, 14-5, 15-3, 15-4, 15-5a
Represent and interpret data.		
3.MD.3	Draw a scaled picture graph and a scaled bar graph to represent a data set with several categories. Solve one- and two-step "how many more" and "how many less" problems using information presented in scaled bar graphs. For example, draw a bar graph in which each square in the bar graph might represent 5 pets.	2-10, 4-6 20-2, 20-3, 20-4, 20-9
3.MD.4	Generate measurement data by measuring lengths using rulers marked with halves and fourths of an inch. Show the data by making a line plot, where the horizontal scale is marked off in appropriate units—whole numbers, halves, or quarters.	14-2, 20-9a
Geometric measurement: understand concepts of area and relate area to multiplication and to addition.		
3.MD.5	Recognize area as an attribute of plane figures and understand concepts of area measurement.	16-5, 16-6a, 16-7d
3.MD.5.a	A square with side length 1 unit, called "a unit square," is said to have "one square unit" of area, and can be used to measure area.	16-5, 16-6b
3.MD.5.b	A plane figure which can be covered without gaps or overlaps by n unit squares is said to have an area of n square units.	16-5, 16-6b
3.MD.6.	Measure areas by counting unit squares (square cm, square m, square in, square ft, and improvised units).	16-5, 16-6a, 16-6,
3.MD.7	Relate area to the operations of multiplication and addition.	16-5, 16-6, 16-8

Standards for Mathematical Content Grade 3		Where to find in enVisionMATH ©2009/2011
3.MD.7.a	Find the area of a rectangle with whole-number side lengths by tiling it, and show that the area is the same as would be found by multiplying the side lengths.	16-5, 16-7a
3.MD.7.b	Multiply side lengths to find areas of rectangles with whole-number side lengths in the context of solving real world and mathematical problems, and represent whole-number products as rectangular areas in mathematical reasoning.	5-2, 16-5, 16-6b, 16-8,
3.MD.7.c	Use tiling to show in a concrete case that the area of a rectangle with whole-number side lengths a and b + c is the sum of a × b and a × c. Use area models to represent the distributive property in mathematical reasoning.	6-1, 6-2, 6-3, 6-4, 16-7a
3.MD.7.d	Recognize area as additive. Find areas of rectilinear figures by decomposing them into non-overlapping rectangles and adding the areas of the non-overlapping parts, applying this technique to solve real world problems.	16-7b, 16-8
Geometric measurement: recognize perimeter as an attribute of plane figures and distinguish between linear and area measures.		
3.MD.8	Solve real world and mathematical problems involving perimeters of polygons, including finding the perimeter given the side lengths, finding an unknown side length, and exhibiting rectangles with the same perimeter and different areas or with the same area and different perimeters.	6-7, 15-5a,16-1, 16-2, 16-3, 16-5

Standards for Mathematical Content Grade 3		Where to find in enVisionMATH ©2009/2011
Geometry		
Reason with shapes and their attributes.		
3.G.1	Understand that shapes in different categories (e.g., rhombuses, rectangles, and others) may share attributes (e.g., having four sides), and that the shared attributes can define a larger category (e.g., quadrilaterals). Recognize rhombuses, rectangles, and squares as examples of quadrilaterals, and draw examples of quadrilaterals that do not belong to any of these subcategories.	10-5, 10-6, 10-7, 10-8a, 10-8b, 10-8
3.G.2	Partition shapes into parts with equal areas. Express the area of each part as a unit fraction of the whole. For example, partition a shape into 4 parts with equal area, and describe the area of each part as 1/4 of the area of the shape.	12-1, 10-8a, 10-8b, 16-7c

Standards for Mathematical Practice
Grade 4

The Standards for Mathematical Practice are an important part of California's Common Core Content Standards. They describe varieties of proficiency that teachers should focus on developing in their students. These practices draw from the NCTM process standards of problem solving, reasoning and proof, communication, representation, and connections and the strands of mathematical proficiency specified in the National Research Council's report *Adding It Up*: adaptive reasoning, strategic competence, conceptual understanding, procedural fluency, and productive disposition.

For each of the Standards for Mathematical Practice presented in the text that follows, is a explanation of the different features and elements of Pearson's *Scott Foresman • Addison Wesley enVisionMATH*™ *California* that help students develop mathematical proficiency.

1 Make sense of problems and persevere in solving them.

Mathematically proficient students start by explaining to themselves the meaning of a problem and looking for entry points to its solution. They analyze givens, constraints, relationships, and goals. They make conjectures about the form and meaning of the solution and plan a solution pathway rather than simply jumping into a solution attempt. They consider analogous problems, and try special cases and simpler forms of the original problem in order to gain insight into its solution. They monitor and evaluate their progress and change course if necessary. Older students might, depending on the context of the problem, transform algebraic expressions or change the viewing window on their graphing calculator to get the information they need. Mathematically proficient students can explain correspondences between equations, verbal descriptions, tables, and graphs or draw diagrams of important features and relationships, graph data, and search for regularity or trends. Younger students might rely on using concrete objects or pictures to help conceptualize and solve a problem. Mathematically proficient students check their answers to problems using a different method, and they continually ask themselves, "Does this make sense? "They can understand the approaches of others to solving complex problems and identify correspondences between different approaches.

Scott Foresman • Addison Wesley enVisionMATH California is built on a foundation of problem-based instruction that has sense-making at its heart. The Problem Solving Handbook, found on pages xviii-xxix, presents to students a 3-phase process that begins with making sense of the problem to solve. The first phase of the process has students ask themselves, *What am I trying to find? What do I know?* to help them identify the givens and constraints of a problem situation. In the second phase, the plan and solve phase, students decide on a solution plan. The Problem-Solving Recording Sheet, a reproducible teaching resource, provides students with a useful structured outline to help them become fluent with thinking about (making sense of) a problem and planning a workable solution pathway.

The structure of each lesson facilitates students' implementation of this process. Every lesson begins with **Problem-Based Interactive Learning,** an activity in which students are presented a problem to solve. They interact with their peers and teachers to make sense of the problem presented and to look for a workable solution. A second feature of each lesson are the Problem Solving exercises for which students persevere to find solutions for each exercise. In each topic is at least one Problem Solving lesson with a primary focus of honing students' sense-making and problem-solving skills.

Throughout the program; for examples, see Grade 4 Lessons 1-3, 1-8, 2-1, 2-2, 2-3, 2-4, 2-5, 2-6, 3-6, 4-6,
5-5, 6-6, 10-6, 10-8, 12-11, 13-10, 14-4, 14-6, 14-9, 14-10, 14-11, 15-5

2 Reason abstractly and quantitatively.

Mathematically proficient students make sense of quantities and their relationships in problem situations. They bring two complementary abilities to bear on problems involving quantitative relationships: the ability to decontextualize—to abstract a given situation and represent it symbolically and manipulate the representing symbols as if they have a life of their own, without necessarily attending to their referents—and the ability to contextualize, to pause as needed during the manipulation process in order to probe into the referents for the symbols involved. Quantitative reasoning entails habits of creating a coherent representation of the problem at hand; considering the units involved; attending to the meaning of quantities, not just how to compute them; and knowing and flexibly using different properties of operations and objects.

Scott Foresman • Addison Wesley enVisionMATH California program provides scaffolded instruction to help students develop both quantitative and abstract reasoning. In the Visual Learning Bridge students learn how to represent the given situation numerically or algebraically. Later in a lesson, students have opportunities to reason abstractly as they endeavor to represent situations symbolically. Throughout the solving process, students are reminded to check back to the problem situation with the **Reasonableness** exercises. In the **Do You Understand** part of the Guided Practice, students gain experiences with quantitative reasoning as they consider the meaning of different parts of an expression or equation. Throughout the exercise sets are Reasoning exercises that focus students' attention on the structure or meaning of an operation rather than the solution.

Throughout the program; for examples, see Grade 4 Lessons 2-6, 5-4, 5-5, 7-3, 7-4, 8-5, 9-2, 10-3, 10-8, 11-3, 12-5, 13-6, 14-11, 15-5.

3 Construct viable arguments and critique the reasoning of others.

Mathematically proficient students understand and use stated assumptions, definitions, and previously established results in constructing arguments. They make conjectures and build a logical progression of statements to explore the truth of their conjectures. They are able to analyze situations by breaking them into cases, and can recognize and use counterexamples. They justify their conclusions, communicate them to others, and respond to the arguments of others. They reason inductively about data, making plausible arguments that take into account the context from which the data arose. Mathematically proficient students are also able to compare the effectiveness of two plausible arguments, distinguish correct logic or reasoning from that which is flawed, and—if there is a flaw in an argument—explain what it is. Elementary students can construct arguments using concrete referents such as objects, drawings, diagrams, and actions. Such arguments can make sense and be correct, even though they are not generalized or made formal until later grades. Later, students learn to determine domains to which an argument applies. Students at all grades can listen or read the arguments of others, decide whether they make sense, and ask useful questions to clarify or improve the arguments.

Consistent with a focus on reasoning and sense-making is a focus on critical reasoning – argumentation and critique of arguments. In Pearson's *Scott Foresman • Addison Wesley enVisionMATH California,* the **Interactive Learning** affords students opportunities to share with classmates their thinking about problems, their solutions, and their reasoning about the solutions. The many **Reasoning** exercises found throughout the program specifically call for students to justify or explain their solutions. The **Writing to Explain** exercises help students develop foundational critical reasoning skills by having them construct explanations for processes. Articulating clearly an explanation for a process is a stepping stone to critical analysis and reasoning of both their own processes and those of others.

Throughout the program; for examples, see Grade 4 Lessons 5-6, 7-5, 11-8, 14-5, 16-11

4 Model with mathematics.

Mathematically proficient students can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace. In early grades, this might be as simple as writing an addition equation to describe a situation. In middle grades, a student might apply proportional reasoning to plan a school event or analyze a problem in the community. By high school, a student might use geometry to solve a design problem or use a function to describe how one quantity of interest depends on another. Mathematically proficient students who can apply what they know are comfortable making assumptions and approximations to simplify a complicated situation, realizing that these may need revision later. They are able to identify important quantities in a practical situation and map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts and formulas. They can analyze those relationships mathematically to draw conclusions. They routinely interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose.

Students in Pearson's *Scott Foresman • Addison Wesley enVisionMATH California,* are introduced to mathematical modeling in the early grades. They first use manipulatives and drawings and then equations to model addition and subtraction situations. The **Visual Learning Bridge** and **Visual Learning Animation** often present real-world situations and students are shown how these can be modeled mathematically. In later years, students expand their modeling skills to include other graphical representations such as tables, graphs, as well as equations.

Throughout the program; for examples, see Grade 4 Lessons 1-1, 1-9, 2-2, 3-3, 3-6, 4-2, 5-5, 8-4, 9-1, 9-5, 10-5, 11-4, 12-1, 12-2, 12-5, 12-6, 12-9, 13-1, 13-2, 13-5, 13-7, 14-11, 15-5, 16-1, 16-2, 16-6

5 Use appropriate tools strategically.

Mathematically proficient students consider the available tools when solving a mathematical problem. These tools might include pencil and paper, concrete models, a ruler, a protractor, a calculator, a spreadsheet, a computer algebra system, a statistical package, or dynamic geometry software. Proficient students are sufficiently familiar with tools appropriate for their grade or course to make sound decisions about when each of these tools might be helpful, recognizing both the insight to be gained and their limitations. For example, mathematically proficient high school students analyze graphs of functions and solutions generated using a graphing calculator. They detect possible errors by strategically using estimation and other mathematical knowledge. When making mathematical models, they know that technology can enable them to visualize the results of varying assumptions, explore consequences, and compare predictions with data. Mathematically proficient students at various grade levels are able to identify relevant external mathematical resources, such as digital content located on a website, and use them to pose or solve problems. They are able to use technological tools to explore and deepen their understanding of concepts.

Students become fluent in the use of a wide assortment of tools ranging from physical objects, including manipulatives, rulers, protractors, and even pencil and paper, to technological tools, such as etools, calculators and computers. As students become more familiar with the tools available to them, they are able to begin making decisions about which tools are more appropriate to solve different kinds of problems.

Throughout the program; for examples, see Grade 4 Lessons 1-2, 1-4, 2-1, 2-5, 3-1, 4-1, 4-6, 5-1, 5-3, 6-1, 6-6, 8-1, 8-3, 9-4, 10-3, 10-6, 11-2, 13-9, 14-1, 14-6, 15-1, 16-5, 16-10

6 Attend to precision.

Mathematically proficient students try to communicate precisely to others. They try to use clear definitions in discussion with others and in their own reasoning. They state the meaning of the symbols they choose, including using the equal sign consistently and appropriately. They are careful about specifying units of measure, and labeling axes to clarify the correspondence with quantities in a problem. They calculate accurately and efficiently, express numerical answers with a degree of precision appropriate for the problem context. In the elementary grades, students give carefully formulated explanations to each other. By the time they reach high school they have learned to examine claims and make explicit use of definitions.

Students are expected to use mathematical terms and symbols with precision. Key terms and concepts are highlighted in each lesson. In the **Do You Understand** feature, students often revisit these key terms and provide explicit definitions or explanations of the terms. For the **Writing to Explain** and **Think About a Process** exercises, students are to provide clear explanations of terms, concepts, or processes and to use new terms accurately and precisely. Students are reminded to use appropriate units of measure when working through solutions and accurate labels on axes when making graphs to represent solutions.

Throughout the program; for examples, see Grade 4 Lessons 6-3, 6-4, 6-5, 8-1, 8-2, 8-3, 9-4, 10-3, 10-6, 11-1, 11-2, 13-9, 14-1, 14-6, 15-1, 16-5, 16-6, 16-10

7 Look for and make use of structure.

Mathematically proficient students look closely to discern a pattern or structure .Young students, for example, might notice that three and seven more is the same amount as seven and three more, or they may sort a collection of shapes according to how many sides the shapes have. Later, students will see 7×8 equals the well remembered $7 \times 5 \times 7 \times 3$, in preparation for learning about the distributive property. In the expression $x^2 + 9x + 14$, older students can see the 14 as 2×7 and the 9 as $2 + 7$.They recognize the significance of an existing line in a geometric figure and can use the strategy of drawing an auxiliary line for solving problems. They also can step back for an overview and shift perspective. They can see complicated things, such as some algebraic expressions, as single objects or as being composed of several objects. For example, they can see $5 \;\; 3(x \;\; y)^2$ as 5 minus a positive number times a square and use that to realize that its value cannot be more than 5 for any real numbers x and y.

Throughout the program, students are encouraged to look for structure as they develop solution plans. In the **Look for a Pattern Problem-Solving** lessons, children in the early years develop a sense of patterning with visual and physical objects.

As students mature in their mathematical thinking, they look for structure in numerical operations by focusing on place value and properties of operations. From this focus on looking for and recognizing structure, students become well-equip to draw from patterns to formalize their thinking about the structure of operations.

Throughout the program; for examples, see Grade 4 Lessons 1-5, 1-6, 2-1, 2-2, 2-3, 3-3, 3-4, 3-5, 4-1, 4-2, 4-3, 4-4, 4-5, 5-1, 5-2, 5-3, 5-4, 5-5, 6-1, 6-2, 6-3, 6-4, 6-5, 7-1, 7-2, 7-3, 7-4, 8-1, 8-4, 9-1, 9-2, 9-4, 9-5, 10-4, 11-1, 11-3, 11-4, 11-7, 12-1, 12-3, 12-4, 12-7, 12-8, 12-9, 13-4, 13-5, 13-6, 13-7, 13-8, 13-9, 14-2, 14-3, 14-4, 14-8, 14-9, 14-10, 16-1, 16-2, 16-7, 16-8, 16-9

8 Look for and express regularity in repeated reasoning.

Mathematically proficient students notice if calculations are repeated, and look both for general methods and for shortcuts. Upper elementary students might notice when dividing 25 by 11 that they are repeating the same calculations over and over again, and conclude they have a repeating decimal. By paying attention to the calculation of slope as they repeatedly check whether points are on the line through (1, 2) with slope 3, middle school students might abstract the equation $(y – 2)/(x – 1) = 3$. Noticing the regularity in the way terms cancel when expanding $(x – 1)(x + 1)$, $(x – 1)(x^2 + x + 1)$, and $(x – 1)(x^3 + x^2 + x + 1)$ might lead them to the general formula for the sum of a geometric series. As they work to solve a problem, mathematically proficient students maintain oversight of the process, while attending to the details. They continually evaluate the reasonableness of their intermediate results.

Once again, throughout the program as a whole, students are prompted to look for repetition in computations to derive shortcuts that can make the problem-solving process more efficient. Students are prompted to think about problems they encountered previously that may share features or processes. They are encouraged to draw on the solution plan developed for that problem, and as their mathematical thinking matures, to look for generalizations that can be applied to other problem situations. The **Interactive Learning** activities offer students opportunities to look for regularity in the way operations behave.

Throughout the program; for examples, see \ Grade 4 Lessons 1-5, 1-7, 4-3, 4-4, 4-5, 5-2, 7-2, 9-2, 9-5.

Correlation of Standards for Mathematical Content

enVisionMATH™ Grade 4

The following shows the alignment of *enVisionMATH* Grade 4 ©2009/2011 to the Common Core State Standards for Grade 4. Included in this correlation are the supplemental lessons that will be available as part of the transitional support that Pearson is providing. These lessons will be available in the summer 2011.

Standards for Mathematical Content Grade 4		Where to find in enVisionMATH ©2009/2011
Operations and Algebraic Thinking		
Use the four operations with whole numbers to solve problems.		
4.OA.1	Interpret a multiplication equation as a comparison, e.g., interpret 35 = 5 × 7 as a statement that 35 is 5 times as many as 7 and 7 times as many as 5. Represent verbal statements of multiplicative comparisons as multiplication equations.	3-1, 3-3, 3-7, 5-8
4.OA.2	Multiply or divide to solve word problems involving multiplicative comparison, e.g., by using drawings and equations with a symbol for the unknown number to represent the problem, distinguishing multiplicative comparison from additive comparison.	3-1, 3-7, 5-8
4.OA.3	Solve multistep word problems posed with whole numbers and having whole-number answers using the four operations, including problems in which remainders must be interpreted. Represent these problems using equations with a letter standing for the unknown quantity. Assess the reasonableness of answers using mental computation and estimation strategies including rounding.	2-1, 2-2, 5-2, 5-4, 6-1, 6-4, 7-2, 7-3a, 7-7, 8-2, 8-3a, 8-3, 8-10, 16-12, 18-1, 18-2, 18-3, 18-5,
Gain familiarity with factors and multiples.		
4.OA.4	Find all factor pairs for a whole number in the range 1–100. Recognize that a whole number is a multiple of each of its factors. Determine whether a given whole number in the range 1–100 is a multiple of a given one-digit number. Determine whether a given whole number in the range 1–100 is prime or composite.	3-2, 3-4, 3-5, 3-6, 8-8, 8-9
Generate and analyze patterns.		
4.OA.5	Generate a number or shape pattern that follows a given rule. Identify apparent features of the pattern that were not explicit in the rule itself. *For example, given the rule "Add 3" and the starting number 1, generate terms in the resulting sequence and observe that the terms appear to alternate between odd and even numbers. Explain informally why the numbers will continue to alternate in this way.*	3-2, 6-2, 6-3, 9-7

Standards for Mathematical Content Grade 4		Where to find in enVisionMATH ©2009/2011
Number and Operations in Base Ten [1]		
Generalize place value understanding for multi-digit whole numbers.		
4.NBT.1	Recognize that in a multi-digit whole number, a digit in one place represents ten times what it represents in the place to its right. *For example, recognize that 700 ÷ 70 = 10 by applying concepts of place value and division.*	1-3a
4.NBT.2	Read and write multi-digit whole numbers using base-ten numerals, number names, and expanded form. Compare two multi-digit numbers based on meanings of the digits in each place, using >, =, and < symbols to record the results of comparisons.	1-1, 1-2, 1-3a, 1-3
4.NBT.3	Use place value understanding to round multi-digit whole numbers to any place.	1-4, 2-2, 5-2, 5-3, 5-4
Use place value understanding and properties of operations to perform multi-digit arithmetic.		
4.NBT.4	Fluently add and subtract multi-digit whole numbers using the standard algorithm.	2-4, 2-5, 2-6, 2-7
4.NBT.5	Multiply a whole number of up to four digits by a one-digit whole number, and multiply two two-digit numbers, using strategies based on place value and the properties of operations. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models.	5-1, 5-2, 5-3, 5-4, 5-5, 5-6, 5-7, 5-8a, 5-8, 7-1, 7-3a, 7-3, 7-4a, 7-4b, 7-4, 7-5,
4.NBT.6	Find whole-number quotients and remainders with up to four-digit dividends and one-digit divisors, using strategies based on place value, the properties of operations, and/or the relationship between multiplication and division. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models.	4-1, 4-2, 4-3, 4-5, 8-1, 8-3a, 8-3b, 8-3c, 8-3, 8-4, 8-5, 8-6, 8-7a, 8-7

[1] *(Grade 4 expectations in this domain are limited to whole numbers less than or equal to 1,000,000.)*

Standards for Mathematical Content Grade 4		Where to find in enVisionMATH ©2009/2011
Number and Operations—Fractions[2]		
Extend understanding of fraction equivalence and ordering.		
4.NF.1	Explain why a fraction *a*/*b* is equivalent to a fraction (*n* × *a*)/(*n* × *b*) by using visual fraction models, with attention to how the number and size of the parts differ even though the two fractions themselves are the same size. Use this principle to recognize and generate equivalent fractions.	10-4, 10-5a, 10-5, 10-9
4.NF.2	Compare two fractions with different numerators and different denominators, e.g., by creating common denominators or numerators, or by comparing to a benchmark fraction such as 1/2. Recognize that comparisons are valid only when the two fractions refer to the same whole. Record the results of comparisons with symbols >, =, or <, and justify the conclusions, e.g., by using a visual fraction model.	10-5a, 10-7, 10-8, 10-9,
Build fractions from unit fractions by applying and extending previous understandings of operations on whole numbers.		
4.NF.3	Understand a fraction *a*/*b* with *a* > 1 as a sum of fractions 1/*b*.	11-1a, 11-1, 11-4
4.NF.3.a	Understand addition and subtraction of fractions as joining and separating parts referring to the same whole.	11-1
4.NF.3.b	Decompose a fraction into a sum of fractions with the same denominator in more than one way, recording each decomposition by an equation. Justify decompositions, e.g., by using a visual fraction model. *Examples: 3/8 = 1/8 + 1/8 + 1/8 ; 3/8 = 1/8 + 2/8 ; 2 1/8 = 1 + 1 + 1/8 = 8/8 + 8/8 + 1/8.*	11-1a, 11-5a
4.NF.3.c	Add and subtract mixed numbers with like denominators, e.g., by replacing each mixed number with an equivalent fraction, and/or by using properties of operations and the relationship between addition and subtraction.	11-5a, 11-5b, 11-5c
4.NF.3.d	Solve word problems involving addition and subtraction of fractions referring to the same whole and having like denominators, e.g., by using visual fraction models and equations to represent the problem.	11-1
4.NF.4	Apply and extend previous understandings of multiplication to multiply a fraction by a whole number.	11-5e
4.NF.4.a	Understand a fraction *a*/*b* as a multiple of 1/*b*. *For example, use a visual fraction model to represent 5/4 as the product 5 × (1/4), recording the conclusion by the equation 5/4 = 5 × (1/4).*	11-5d
4.NF.4.b	Understand a multiple of *a*/*b* as a multiple of 1/*b*, and use this	11-5e, 11-5f

[2] (Grade 4 expectations in this domain are limited to fractions with denominators 2, 3, 4, 5, 6, 8, 10, 12, and 100.)

	understanding to multiply a fraction by a whole number. *For example, use a visual fraction model to express 3 × (2/5) as 6 × (1/5), recognizing this product as 6/5. (In general, n × (a/b) = (n × a)/b.)*	
4.NF.4.c	Solve word problems involving multiplication of a fraction by a whole number, e.g., by using visual fraction models and equations to represent the problem. *For example, if each person at a party will eat 3/8 of a pound of roast beef, and there will be 5 people at the party, how many pounds of roast beef will be needed? Between what two whole numbers does your answer lie?*	11-5f
Understand decimal notation for fractions, and compare decimal fractions.		
4.NF.5	Express a fraction with denominator 10 as an equivalent fraction with denominator 100, and use this technique to add two fractions with respective denominators 10 and 100. *For example, express 3/10 as 30/100, and add 3/10 + 4/100 = 34/100.* (*Students who can generate equivalent fractions can develop strategies for adding fractions with unlike denominators in general. But addition and subtraction with unlike denominators in general is not a requirement at this grade.*)	12-3, 12-4, 12-5a
4.NF.6	Use decimal notation for fractions with denominators 10 or 100. *For example, rewrite 0.62 as 62/100; describe a length as 0.62 meters; locate 0.62 on a number line diagram.*	12-1, 12-3, 12-5a
4.NF.7	Compare two decimals to hundredths by reasoning about their size. Recognize that comparisons are valid only when the two decimals refer to the same whole. Record the results of comparisons with the symbols >, =, or <, and justify the conclusions, e.g., by using a visual model.	12-2

Standards for Mathematical Content Grade 4		Where to find in enVisionMATH ©2009/2011
Measurement and Data		
Solve problems involving measurement and conversion of measurements from a larger unit to a smaller unit.		
4.MD.1	Know relative sizes of measurement units within one system of units including km, m, cm; kg, g; lb, oz.; l, ml; hr, min, sec. Within a single system of measurement, express measurements in a larger unit in terms of a smaller unit. Record measurement equivalents in a two-column table. *For example, know that 1 ft is 12 times as long as 1 in. Express the length of a 4 ft snake as 48 in. Generate a conversion table for feet and inches listing the number pairs (1, 12), (2, 24), (3, 36), …*	16-1, 16-3, 16-4, 16-5, 16-6, 16-7, 16-8, 16-9
4.MD.2	Use the four operations to solve word problems involving distances, intervals of time, liquid volumes, masses of objects, and money, including problems involving simple fractions or decimals, and problems that require expressing measurements given in a larger unit in terms of a smaller unit. Represent measurement quantities using diagrams such as number line diagrams that feature a measurement scale.	1-7a, 11-4, 12-6, 13-7, 16-4, 16-8, 16-9, 16-12a, 16-12
4.MD.3	Apply the area and perimeter formulas for rectangles in real world and mathematical problems. *For example, find the width of a rectangular room given the area of the flooring and the length, by viewing the area formula as a multiplication equation with an unknown factor.*	14-2, 14-6, 14-7a
Represent and interpret data.		
4.MD.4	Make a line plot to display a data set of measurements in fractions of a unit (1/2, ¼, 1/8). Solve problems involving addition and subtraction of fractions by using information presented in line plots. *For example, from a line plot find and interpret the difference in length between the longest and shortest specimens in an insect collection.*	16-12b
Geometric measurement: understand concepts of angle and measure angles.		
4.MD.5	Recognize angles as geometric shapes that are formed wherever two rays share a common endpoint, and understand concepts of angle measurement:	9-2, 9-3a, 9-3
4.MD.5.a	An angle is measured with reference to a circle with its center at the common endpoint of the rays, by considering the fraction of the circular arc between the points where the two rays intersect the circle. An angle that turns through 1/360 of a circle is called a “one-degree angle,” and can be used to measure angles.	9-3a, 9-3b, 9-3
4.MD.5.b	An angle that turns through *n* one-degree angles is said to have an angle measure of *n* degrees.	9-3b, 9-4a

4.MD.6	Measure angles in whole-number degrees using a protractor. Sketch angles of specified measure.	9-3, 9-4a
4.MD.7	Recognize angle measure as additive. When an angle is decomposed into non-overlapping parts, the angle measure of the whole is the sum of the angle measures of the parts. Solve addition and subtraction problems to find unknown angles on a diagram in real world and mathematical problems, e.g., by using an equation with a symbol for the unknown angle measure.	9-4a

Standards for Mathematical Content Grade 4		Where to find in enVisionMATH ©2009/2011
Geometry		
Draw and identify lines and angles, and classify shapes by properties of their lines and angles.		
4.G.1	Draw points, lines, line segments, rays, angles (right, acute, obtuse), and perpendicular and parallel lines. Identify these in two-dimensional figures.	9-1, 9-2, 9-3a, 9-3b
4.G.2	Classify two-dimensional figures based on the presence or absence of parallel or perpendicular lines, or the presence or absence of angles of a specified size. Recognize right triangles as a category, and identify right triangles.	9-4, 9-5, 9-6, 9-7
4.G.3	Recognize a line of symmetry for a two-dimensional figure as a line across the figure such that the figure can be folded along the line into matching parts. Identify line-symmetric figures and draw lines of symmetry.	19-5

Common Core State Standards
Standards for Mathematical Practice
Grade 5

The Standards for Mathematical Practice are an important part of the Common Core State Standards. They describe varieties of proficiency that teachers should focus on developing in their students. These practices draw from the NCTM process standards of problem solving, reasoning and proof, communication, representation, and connections and the strands of mathematical proficiency specified in the National Research Council's report *Adding It Up*: adaptive reasoning, strategic competence, conceptual understanding, procedural fluency, and productive disposition.

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1. Make sense of problems and persevere in solving them.

Mathematically proficient students start by explaining to themselves the meaning of a problem and looking for entry points to its solution. They analyze givens, constraints, relationships, and goals. They make conjectures about the form and meaning of the solution and plan a solution pathway rather than simply jumping into a solution attempt. They consider analogous problems, and try special cases and simpler forms of the original problem in order to gain insight into its solution. They monitor and evaluate their progress and change course if necessary. Older students might, depending on the context of the problem, transform algebraic expressions or change the viewing window on their graphing calculator to get the information they need. Mathematically proficient students can explain correspondences between equations, verbal descriptions, tables, and graphs or draw diagrams of important features and relationships, graph data, and search for regularity or trends. Younger students might rely on using concrete objects or pictures to help conceptualize and solve a problem. Mathematically proficient students check their answers to problems using a different method, and they continually ask themselves, "Does this make sense? "They can understand the approaches of others to solving complex problems and identify correspondences between different approaches.

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Throughout the program; for examples, see *enVisionMATH* Grade 5 Lessons 1-6, 2-4, 2-7, 3-10, 4-3, 4-7, 5-8, 7-7,
8-9, 9-3, 11-11

2 Reason abstractly and quantitatively.

Mathematically proficient students make sense of quantities and their relationships in problem situations. They bring two complementary abilities to bear on problems involving quantitative relationships: the ability to decontextualize—to abstract a given situation and represent it symbolically and manipulate the representing symbols as if they have a life of their own, without necessarily attending to their referents—and the ability to contextualize, to pause as needed during the manipulation process in order to probe into the referents for the symbols involved. Quantitative reasoning entails habits of creating a coherent representation of the problem at hand; considering the units involved; attending to the meaning of quantities, not just how to compute them; and knowing and flexibly using different properties of operations and objects.

Scott Foresman • Addison Wesley enVisionMATH Program provides scaffolded instruction to help students develop both quantitative and abstract reasoning. In the Visual Learning Bridge students learn how to represent the given situation numerically or algebraically. Later in a lesson, students have opportunities to reason abstractly as they endeavor to represent situations symbolically. Throughout the solving process, students are reminded to check back to the problem situation with the **Reasonableness** exercises. In the **Do You Understand** part of the Guided Practice, students gain experiences with quantitative reasoning as they consider the meaning of different parts of an expression or equation. Throughout the exercise sets are Reasoning exercises that focus students' attention on the structure or meaning of an operation rather than the solution.

Throughout the program; for examples, see *enVisionMATH* Grade 5 Lessons 1-6, 3-1, 6-3, 15-3

3 Construct viable arguments and critique the reasoning of others.

Mathematically proficient students understand and use stated assumptions, definitions, and previously established results in constructing arguments. They make conjectures and build a logical progression of statements to explore the truth of their conjectures. They are able to analyze situations by breaking them into cases, and can recognize and use counterexamples. They justify their conclusions, communicate them to others, and respond to the arguments of others. They reason inductively about data, making plausible arguments that take into account the context from which the data arose. Mathematically proficient students are also able to compare the effectiveness of two plausible arguments, distinguish correct logic or reasoning from that which is flawed, and—if there is a flaw in an argument—explain what it is. Elementary students can construct arguments using concrete referents such as objects, drawings, diagrams, and actions. Such arguments can make sense and be correct, even though they are not generalized or made formal until later grades. Later, students learn to determine domains to which an argument applies. Students at all grades can listen or read the arguments of others, decide whether they make sense, and ask useful questions to clarify or improve the arguments.

Consistent with a focus on reasoning and sense-making is a focus on critical reasoning – argumentation and critique of arguments. In Pearson's *Scott Foresman • Addison Wesley enVisionMATH,* the Problem-Based Interactive Learning affords students opportunities to share with classmates their thinking about problems, their solutions, and their reasoning about the solutions. The many Reasoning exercises found throughout the program specifically call for students to justify or explain their solutions. The **Writing to Explain** exercises help students develop foundational critical reasoning skills by having them construct explanations for processes. Articulating clearly an explanation for a process is a stepping stone to critical analysis and reasoning of both their own processes and those of others.

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4 Model with mathematics.

Mathematically proficient students can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace. In early grades, this might be as simple as writing an addition equation to describe a situation. In middle grades, a student might apply proportional reasoning to plan a school event or analyze a problem in the community. By high school, a student might use geometry to solve a design problem or use a function to describe how one quantity of interest depends on another. Mathematically proficient students who can apply what they know are comfortable making assumptions and approximations to simplify a complicated situation, realizing that these may need revision later. They are able to identify important quantities in a practical situation and map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts and formulas. They can analyze those relationships mathematically to draw conclusions. They routinely interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose.

Students in Pearson's *Scott Foresman • Addison Wesley enVisionMATH,* are introduced to mathematical modeling in the early grades. They first use manipulatives and drawings and then equations to model addition and subtraction situations. The Visual Learning Bridge and Visual Learning Animation often present real-world situations and students are shown how these can be modeled mathematically. In later years, students expand their modeling skills to include other graphical representations such as tables, graphs, as well as equations.

Throughout the program; for examples, see *enVisionMATH* Grade 5 Lessons 1-4, 2-4, 4-6, 8-9, 11-4, 11-11, 14-4, 15-2, 15-5

5 Use appropriate tools strategically.

Mathematically proficient students consider the available tools when solving a mathematical problem. These tools might include pencil and paper, concrete models, a ruler, a protractor, a calculator, a spreadsheet, a computer algebra system, a statistical package, or dynamic geometry software. Proficient students are sufficiently familiar with tools appropriate for their grade or course to make sound decisions about when each of these tools might be helpful, recognizing both the insight to be gained and their limitations. For example, mathematically proficient high school students analyze graphs of functions and solutions generated using a graphing calculator. They detect possible errors by strategically using estimation and other mathematical knowledge. When making mathematical models, they know that technology can enable them to visualize the results of varying assumptions, explore consequences, and compare predictions with data. Mathematically proficient students at various grade levels are able to identify relevant external mathematical resources, such as digital content located on a website, and use them to pose or solve problems. They are able to use technological tools to explore and deepen their understanding of concepts.

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6 Attend to precision.

Mathematically proficient students try to communicate precisely to others. They try to use clear definitions in discussion with others and in their own reasoning. They state the meaning of the symbols they choose, including using the equal sign consistently and appropriately. They are careful about specifying units of measure, and labeling axes to clarify the correspondence with quantities in a problem. They calculate accurately and efficiently, express numerical answers with a degree of precision appropriate for the problem context. In the elementary grades, students give carefully formulated explanations to each other. By the time they reach high school they have learned to examine claims and make explicit use of definitions.

Students are expected to use mathematical terms and symbols with precision. Key terms and concepts are highlighted in each lesson. In the **Do You Understand** feature, students often revisit these key terms and provide explicit definitions or explanations of the terms. For the Writing to Explain and Think About a Process exercises, students are to provide clear explanations of terms, concepts, or processes and to use new terms accurately and precisely. Students are reminded to use appropriate units of measure when working through solutions and accurate labels on axes when making graphs to represent solutions.

Throughout the program; for examples, see *enVisionMATH* Grade 5 Lessons 2-4, 3-3, 4-6, 8-8, 9-2, 14-2 9-11, 14-6, 15-3, 16-1

7 Look for and make use of structure.

Mathematically proficient students look closely to discern a pattern or structure .Young students, for example, might notice that three and seven more is the same amount as seven and three more, or they may sort a collection of shapes according to how many sides the shapes have. Later, students will see 7×8 equals the well remembered $7 \times 5 \times 7 \times 3$, in preparation for learning about the distributive property. In the expression $x^2 + 9x + 14$, older students can see the 14 as 2×7 and the 9 as $2 + 7$.They recognize the significance of an existing line in a geometric figure and can use the strategy of drawing an auxiliary line for solving problems. They also can step back for an overview and shift perspective. They can see complicated things, such as some algebraic expressions, as single objects or as being composed of several objects. For example, they can see $5 \; 3(x \; y)^2$ as 5 minus a positive number times a square and use that to realize that its value cannot be more than 5 for any real numbers x and y.

Throughout the program, students are encouraged to look for structure as they develop solution plans. In the **Look for a Pattern Problem-Solving** lessons, children in the early years develop a sense of patterning with visual and physical objects.

As students mature in their mathematical thinking, they look for structure in numerical operations by focusing on place value and properties of operations. From this focus on looking for and recognizing structure, students become well-equip to draw from patterns to formalize their thinking about the structure of operations.

Throughout the program; for examples, see *enVisionMATH* Grade 5 Lessons 2-1, 2-6, 3-2, 4-4, 8-9, 11-9, 15-6

8 Look for and express regularity in repeated reasoning.

Mathematically proficient students notice if calculations are repeated, and look both for general methods and for shortcuts. Upper elementary students might notice when dividing 25 by 11 that they are repeating the same calculations over and over again, and conclude they have a repeating decimal. By paying attention to the calculation of slope as they repeatedly check whether points are on the line through (1, 2) with slope 3, middle school students might abstract the equation $(y - 2)/(x - 1) = 3$. Noticing the regularity in the way terms cancel when expanding $(x - 1)(x + 1)$, $(x - 1)(x^2 + x + 1)$, and $(x - 1)(x^3 + x^2 + x + 1)$ might lead them to the general formula for the sum of a geometric series. As they work to solve a problem, mathematically proficient students maintain oversight of the process, while attending to the details. They continually evaluate the reasonableness of their intermediate results.

Once again, throughout the program as a whole, students are prompted to look for repetition in computations to look for shortcuts that can make the problem-solving process more efficient. Students are prompted to think about problems they encountered previously that may share features or processes. They are encouraged to draw on the solution plan developed for that problem, and as their mathematical thinking matures, to look for generalizations that can be applied to other problem situations. The **Problem-Based Interactive Learning** activities offer students opportunities to look for regularity in the way operations behave.

Throughout the program; for examples, see *enVisionMATH* Grade 5 Lessons 2-2, 3-9, 7-2, 7-3, 7-7, 9-4, 9-5, 11-1, 12-3, 13-4, 14-7

Correlation of Standards for Mathematical Content

enVisionMATH® Grade 5

The following shows the alignment of *enVisionMATH* Grade 5 ©2009/2011 to the Common Core State Standards for Grade 5. Included in this correlation are the supplemental lessons that will be available as part of the transitional support that Pearson is providing. These lessons will be available in the summer 2011.

Standards for Mathematical Content Grade 5		Where to find in *enVisionMATH* ©2009/2011
Operations and Algebraic Thinking		
Write and interpret numerical expressions.		
5.OA.1	Use parentheses, brackets, or braces in numerical expressions, and evaluate expressions with these symbols.	6-4, 6-5, 6-6a
5.OA.2	Write simple expressions that record calculations with numbers, and interpret numerical expressions without evaluating them.	3-8, 6-1, 6-3
Analyze patterns and relationships.		
5.OA.3	Generate two numerical patterns using two given rules. Identify apparent relationships between corresponding terms. Form ordered pairs consisting of corresponding terms from the two patterns, and graph the ordered pairs on a coordinate plane.	6-4a, 6-6b, 6-6c, 17-4b, 17-4c

Standards for Mathematical Content Grade 5		Where to find in *enVisionMATH* ©2009/2011
Number and Operations in Base Ten		
Understand the place value system.		
5.NBT.1	Recognize that in a multi-digit number, a digit in one place represents 10 times as much as it represents in the place to its right and 1/10 of what it represents in the place to its left.	7-1
5.NBT.2	Explain patterns in the number of zeros of the product when multiplying a number by powers of 10, and explain patterns in the placement of the decimal point when a decimal is multiplied or divided by a power of 10. Use whole-number exponents to denote powers of 10.	7-1, 7-5
5.NBT.3	Read, write, and compare decimals to thousandths.	1-3, 1-4
5.NBT.3.a	Read and write decimals to thousandths using base-ten numerals, number names, and expanded form, e.g., $347.392 = 3 \times 100 + 4 \times 10 + 7 \times 1 + 3 \times (1/10) + 9 \times (1/100) + 2 \times (1/1000)$.	1-3, 9-8, 9-9
5.NBT.3.b	Compare two decimals to thousandths based on meanings of the digits in each place, using >, =, and < symbols to record the results of comparisons.	1-4
5.NBT.4	Use place value understanding to round decimals to any place.	2-2
Perform operations with multi-digit whole numbers and with decimals to hundredths.		
5.NBT.5	Fluently multiply multi-digit whole numbers using the standard algorithm.	3-4, 3-5, 3-6, 3-8, 5-3, 5-8, 6-4, 10-7
5.NBT.6	Find whole-number quotients of whole numbers with up to four-digit dividends and two-digit divisors, using strategies based on place value, the properties of operations, and/or the relationship between multiplication and division. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models.	4-1, 4-2, 4-3, 4-4, 4-5, 4-6, 5-1, 5-2, 5-3a, 5-3, 5-4, 5-5, 5-6, 5-7, 5-8
5.NBT.7	Add, subtract, multiply, and divide decimals to hundredths, using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; relate the strategy to a written method and explain the reasoning used.	2-6a, 2-6, 2-7, 2-8, 7-2, 7-3, 7-4a, 7-4b, 7-4, 7-5, 7-6a, 7-6, 7-7, 7-8

Standards for Mathematical Content Grade 5		Where to find in *enVisionMATH* ©2009/2011
Number and Operations—Fractions		
Use equivalent fractions as a strategy to add and subtract fractions.		
5.NF.1	Add and subtract fractions with unlike denominators (including mixed numbers) by replacing given fractions with equivalent fractions in such a way as to produce an equivalent sum or difference of fractions with like denominators.	10-3, 10-4, 10-5a, 10-5, 10-6, 10-7a
5.NF.2	Solve word problems involving addition and subtraction of fractions referring to the same whole, including cases of unlike denominators, e.g., by using visual fraction models or equations to represent the problem. Use benchmark fractions and number sense of fractions to estimate mentally and assess the reasonableness of answers.	9-7, 9-11, 10-1a, 10-1, 10-2, 10-3, 10-4, 10-5a, 10-5, 10-6, 10-7a
Apply and extend previous understandings of multiplication and division to multiply and divide fractions.		
5.NF.3	Interpret a fraction as division of the numerator by the denominator ($a/b = a \div b$). Solve word problems involving division of whole numbers leading to answers in the form of fractions or mixed numbers, e.g., by using visual fraction models or equations to represent the problem.	9-2
5.NF.4	Apply and extend previous understandings of multiplication to multiply a fraction or whole number by a fraction.	11-1, 11-2, 11-3
5.NF.4.a	Interpret the product $(a/b) \times q$ as a parts of a partition of q into b equal parts; equivalently, as the result of a sequence of operations $a \times q \div b$.	11-1, 11-2, 11-3
5.NF.4.b	Find the area of a rectangle with fractional side lengths by tiling it with unit squares of the appropriate unit fraction side lengths, and show that the area is the same as would be found by multiplying the side lengths. Multiply fractional side lengths to find areas of rectangles, and represent fraction products as rectangular areas.	11-3a
5.NF.5	Interpret multiplication as scaling (resizing), by:	11-4a
5.NF.5.a	Comparing the size of a product to the size of one factor on the basis of the size of the other factor, without performing the indicated multiplication.	11-2a
5.NF.5.b	Explaining why multiplying a given number by a fraction greater than 1 results in a product greater than the given number (recognizing multiplication by whole numbers greater than 1 as a familiar case); explaining why multiplying a given number by a fraction less than 1 results in a product smaller than the given number; and relating the principle of fraction equivalence	11-4a

Standards for Mathematical Content Grade 5		Where to find in *enVisionMATH* ©2009/2011
	$a/b = (n \times a)/(n \times b)$ to the effect of multiplying a/b by 1.	
5.NF.6	Solve real world problems involving multiplication of fractions and mixed numbers, e.g., by using visual fraction models or equations to represent the problem.	11-1, 11-2, 11-3
5.NF.7	Apply and extend previous understandings of division to divide unit fractions by whole numbers and whole numbers by unit fractions. (*Students able to multiply fractions in general can develop strategies to divide fractions in general, by reasoning about the relationship between multiplication and division. But division of a fraction by a fraction is not a requirement at this grade.*)	11-4, 11-5a
5.NF.7.a	Interpret division of a unit fraction by a non-zero whole number, and compute such quotients.	11-5a
5.NF.7.b	Interpret division of a whole number by a unit fraction, and compute such quotients.	11-4
5.NF.7.c	Solve real world problems involving division of unit fractions by non-zero whole numbers and division of whole numbers by unit fractions, e.g., by using visual fraction models and equations to represent the problem.	11-4, 11-5a

Standards for Mathematical Content Grade 5		Where to find in *enVisionMATH* ©2009/2011
Measurement and Data		
Convert like measurement units within a given measurement system.		
5.MD.1	Convert among different-sized standard measurement units within a given measurement system (e.g., convert 5 cm to 0.05 m), and use these conversions in solving multi-step, real world problems.	14-1, 14-2, 14-3, 14-4, 14-5
Represent and interpret data.		
5.MD.2	Make a line plot to display a data set of measurements in fractions of a unit (1/2, 1/4, 1/8). Use operations on fractions for this grade to solve problems involving information presented in line plots.	18-3a, 18-3b
Geometric measurement: understand concepts of volume and relate volume to multiplication and to addition.		
5.MD.3	Recognize volume as an attribute of solid figures and understand concepts of volume measurement.	13-5a, 13-5
5.MD.3.a	A cube with side length 1 unit, called a “unit cube,” is said to have “one cubic unit” of volume, and can be used to measure volume.	13-5a, 13-5
5.MD.3.b	A solid figure which can be packed without gaps or overlaps using n unit cubes is said to have a volume of n cubic units.	13-5a, 13-5
5.MD.4	Measure volumes by counting unit cubes, using cubic cm, cubic in, cubic ft, and improvised units.	13-5a
5.MD.5	Relate volume to the operations of multiplication and addition and solve real world and mathematical problems involving volume.	13-5, 13-6
5.MD.5.a	Find the volume of a right rectangular prism with whole-number side lengths by packing it with unit cubes, and show that the volume is the same as would be found by multiplying the edge lengths, equivalently by multiplying the height by the area of the base. Represent threefold whole-number products as volumes, e.g., to represent the associative property of multiplication.	13-5a, 13-5
5.MD.5.b	Apply the formulas $V = l \times w \times h$ and $V = b \times h$ for rectangular prisms to find volumes of right rectangular prisms with whole-number edge lengths in the context of solving real world and mathematical problems.	13-5, 13-6a
5.MD.5.c	Recognize volume as additive. Find volumes of solid figures composed of two non-overlapping right rectangular prisms by adding the volumes of the non-overlapping parts, applying this technique to solve real world problems.	13-6a, 13-6

Standards for Mathematical Content Grade 5		Where to find in *enVisionMATH* ©2009/2011
Geometry		
Graph points on the coordinate plane to solve real-world and mathematical problems.		
5.G.1	Use a pair of perpendicular number lines, called axes, to define a coordinate system, with the intersection of the lines (the origin) arranged to coincide with the 0 on each line and a given point in the plane located by using an ordered pair of numbers, called its coordinates. Understand that the first number indicates how far to travel from the origin in the direction of one axis, and the second number indicates how far to travel in the direction of the second axis, with the convention that the names of the two axes and the coordinates correspond (e.g., x-axis and x-coordinate, y-axis and y-coordinate).	17-2, 17-3, 17-4a, 17-4b, 17-4c
5.G.2	Represent real world and mathematical problems by graphing points in the first quadrant of the coordinate plane, and interpret coordinate values of points in the context of the situation.	17-2, 17-4b, 17-4c
Classify two-dimensional figures into categories based on their properties.		
5.G.3	Understand that attributes belonging to a category of two-dimensional figures also belong to all subcategories of that category.	8-3, 8-4, 8-5, 8-6a, 8-6
5.G.4	Classify two-dimensional figures in a hierarchy based on properties.	8-3, 8-4, 8-5, 8-6a, 8-6b, 8-6

Common Core State Standards

Standards for Mathematical Practice
Grade 6

The Standards for Mathematical Practice are an important part of the Common Core State Standards. They describe varieties of proficiency that teachers should focus on developing in their students. These practices draw from the NCTM process standards of problem solving, reasoning and proof, communication, representation, and connections and the strands of mathematical proficiency specified in the National Research Council's report *Adding It Up*: adaptive reasoning, strategic competence, conceptual understanding, procedural fluency, and productive disposition.

For each of the Standards for Mathematical Practice presented in the text that follows, is a explanation of the different features and elements of Pearson's *Scott Foresman • Addison Wesley enVisionMATH*™ that help students develop mathematical proficiency.

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1. Make sense of problems and persevere in solving them.

Mathematically proficient students start by explaining to themselves the meaning of a problem and looking for entry points to its solution. They analyze givens, constraints, relationships, and goals. They make conjectures about the form and meaning of the solution and plan a solution pathway rather than simply jumping into a solution attempt. They consider analogous problems, and try special cases and simpler forms of the original problem in order to gain insight into its solution. They monitor and evaluate their progress and change course if necessary. Older students might, depending on the context of the problem, transform algebraic expressions or change the viewing window on their graphing calculator to get the information they need. Mathematically proficient students can explain correspondences between equations, verbal descriptions, tables, and graphs or draw diagrams of important features and relationships, graph data, and search for regularity or trends. Younger students might rely on using concrete objects or pictures to help conceptualize and solve a problem. Mathematically proficient students check their answers to problems using a different method, and they continually ask themselves, "Does this make sense? "They can understand the approaches of others to solving complex problems and identify correspondences between different approaches.

Scott Foresman • Addison Wesley enVisionMATH is built on a foundation of problem-based instruction that has sense-making at its heart. The Problem Solving Handbook, found on pages xviii-xxix, presents to students a 3-phase process that begins with making sense of the problem to solve. The first phase of the process has students ask themselves, "What am I trying to find? What do I know?" to help them identify the givens and constraints of a problem situation. In the second phase, the plan and solve phase, students decide on a solution plan. The Problem-Solving Recording Sheet, a reproducible teaching resource, provides students with a useful structured outline to help them become fluent with thinking about (making sense of) a problem and planning a workable solution pathway.

The structure of each lesson facilitates students' implementation of this process. Every lesson begins with **Problem-Based Interactive Learning,** an activity in which students are presented a problem to solve. They interact with their peers and teachers to make sense of the problem presented and to look for a workable solution. A second feature of each lesson are the Problem

Solving exercises for which students persevere to find solutions for each exercise. In each topic is at least one Problem Solving lesson with a primary focus of honing students' sense-making and problem-solving skills .

For examples, see *enVisionMATH* Grade 6 Lessons 1-7, 2-8, 3-10, 4-3, 4-5, 5-7, 6-5, 7-7, 8-5, 9-7, 10-10, 11-9, 12-6, 13-4, 14-7, 15-7, 16-6, 17-6, 18-5, 19-4, 19-11

2. Reason abstractly and quantitatively.

Mathematically proficient students make sense of quantities and their relationships in problem situations. They bring two complementary abilities to bear on problems involving quantitative relationships: the ability to decontextualize—to abstract a given situation and represent it symbolically and manipulate the representing symbols as if they have a life of their own, without necessarily attending to their referents—and the ability to contextualize, to pause as needed during the manipulation process in order to probe into the referents for the symbols involved. Quantitative reasoning entails habits of creating a coherent representation of the problem at hand; considering the units involved; attending to the meaning of quantities, not just how to compute them; and knowing and flexibly using different properties of operations and objects.

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3. Construct viable arguments and critique the reasoning of others.

Mathematically proficient students understand and use stated assumptions, definitions, and previously established results in constructing arguments. They make conjectures and build a logical progression of statements to explore the truth of their conjectures. They are able to analyze situations by breaking them into cases, and can recognize and use counterexamples. They justify their conclusions, communicate them to others, and respond to the arguments of others. They reason inductively about data, making plausible arguments that take into account the context from which the data arose. Mathematically proficient students are also able to compare the effectiveness of two plausible arguments, distinguish correct logic or reasoning from that which is flawed, and—if there is a flaw in an argument—explain what it is. Elementary students can construct arguments using concrete referents such as objects, drawings, diagrams, and actions. Such arguments can make sense and be correct, even though they are not generalized or made formal until later grades. Later, students learn to determine domains to which an argument applies. Students at all grades can listen or read the arguments of others, decide whether they make sense, and ask useful questions to clarify or improve the arguments.

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4. Model with mathematics.

Mathematically proficient students can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace. In early grades, this might be as simple as writing an addition equation to describe a situation. In middle grades, a student might apply proportional reasoning to plan a school event or analyze a problem in the community. By high school, a student might use geometry to solve a design problem or use a function to describe how one quantity of interest depends on another. Mathematically proficient students who can apply what they know are comfortable making assumptions and approximations to simplify a complicated situation, realizing that these may need revision later. They are able to identify important quantities in a practical situation and map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts and formulas. They can analyze those relationships mathematically to draw conclusions. They routinely interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose.

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For examples, see *enVisionMATH* Grade 6 Lessons 2-8, 3-5, 4-2, 5-5, 8-3, 9-1, 14-1, 15-3, 17-5, 17-6, 18-5

5. Use appropriate tools strategically.

Mathematically proficient students consider the available tools when solving a mathematical problem. These tools might include pencil and paper, concrete models, a ruler, a protractor, a calculator, a spreadsheet, a computer algebra system, a statistical package, or dynamic geometry software. Proficient students are sufficiently familiar with tools appropriate for their grade or course to make sound decisions about when each of these tools might be helpful, recognizing both the insight to be gained and their limitations. For example, mathematically proficient high school students analyze graphs of functions and solutions generated using a graphing calculator. They detect possible errors by strategically using estimation and other mathematical knowledge. When making mathematical models, they know that technology can enable them to visualize the results of varying assumptions, explore consequences, and compare predictions with data. Mathematically proficient students at various grade levels are able to identify relevant external mathematical resources, such as digital content located on a website, and use them to pose or solve problems. They are able to use technological tools to explore and deepen their understanding of concepts.

Students become fluent in the use of a wide assortment of tools ranging from physical objects, including manipulatives, rulers, protractors, and even pencil and paper, to technological tools, such as etools, calculators and computers. As students become more familiar with the tools available to them, they are able to begin making decisions about which tools are more appropriate to solve different kinds of problems.

For examples, see *enVisionMATH* Grade 6 Lessons 2-3, 4-3, 8-3, 10-3, 11-3, 12-5, 13-5, 14-6, 15-5, 17-5, 19-2, 20-4

6. Attend to precision.

Mathematically proficient students try to communicate precisely to others. They try to use clear definitions in discussion with others and in their own reasoning. They state the meaning of the symbols they choose, including using the equal sign consistently and appropriately. They are careful about specifying units of measure, and labeling axes to clarify the correspondence with quantities in a problem. They calculate accurately and efficiently, express numerical answers with a degree of precision appropriate for the problem context. In the elementary grades, students give carefully formulated explanations to each other. By the time they reach high school they have learned to examine claims and make explicit use of definitions.

Students are expected to use mathematical terms and symbols with precision. Key terms and concepts are highlighted in each lesson. In the **Do You Understand** feature, students often revisit these key terms and provide explicit definitions or explanations of the terms. For the Writing to Explain and Think About a Process exercises, students are to provide clear explanations of terms, concepts, or processes and to use new terms accurately and precisely. Students are reminded to use appropriate units of measure when working through solutions and accurate labels on axes when making graphs to represent solutions.

For examples, see *enVisionMATH* Grade 6 Lessons 2-4, 3-1, 4-1, 6-2, 7-2, 10-6, 11-3, 13-4, 14-6, 15-2, 17-2, 17-3

7. Look for and make use of structure.

Mathematically proficient students look closely to discern a pattern or structure .Young students, for example, might notice that three and seven more is the same amount as seven and three more, or they may sort a collection of shapes according to how many sides the shapes have. Later, students will see 7×8 equals the well remembered $7 \times 5 \times 7 \times 3$, in preparation for learning about the distributive property. In the expression $x^2 + 9x + 14$, older students can see the 14 as 2×7 and the 9 as $2 + 7$.They recognize the significance of an existing line in a geometric figure and can use the strategy of drawing an auxiliary line for solving problems. They also can step back for an overview and shift perspective. They can see complicated things, such as some algebraic expressions, as single objects or as being composed of several objects. For example, they can see $5 \quad 3(x \quad y)^2$ as 5 minus a positive number times a square and use that to realize that its value cannot be more than 5 for any real numbers x and y.

Throughout the program, students are encouraged to look for structure as they develop solution plans. In the **Look for a Pattern Problem-Solving** lessons, children in the early years develop a sense of patterning with visual and physical objects.

As students mature in their mathematical thinking, they look for structure in numerical operations by focusing on place value and properties of operations. From this focus on looking for and recognizing structure, students become well-equip to draw from patterns to formalize their thinking about the structure of operations.

For examples, see *enVisionMATH* Grade 6 Lessons 2-2, 2-3, 2-4, 4-1, 6-2, 8-4, 9-2, 10-6, 11-5, 15-2, 16-2

8 Look for and express regularity in repeated reasoning.

Mathematically proficient students notice if calculations are repeated, and look both for general methods and for shortcuts. Upper elementary students might notice when dividing 25 by 11 that they are repeating the same calculations over and over again, and conclude they have a repeating decimal. By paying attention to the calculation of slope as they repeatedly check whether points are on the line through (1, 2) with slope 3, middle school students might abstract the equation $(y – 2)/(x – 1) = 3$. Noticing the regularity in the way terms cancel when expanding $(x – 1)(x + 1)$, $(x – 1)(x^2 + x + 1)$, and $(x – 1)(x^3 + x^2 + x + 1)$ might lead them to the general formula for the sum of a geometric series. As they work to solve a problem, mathematically proficient students maintain oversight of the process, while attending to the details. They continually evaluate the reasonableness of their intermediate results.

Once again, throughout the program as a whole, students are prompted to look for repetition in computations to derive shortcuts that can make the problem-solving process more efficient. Students are prompted to think about problems they encountered previously that may share features or processes. They are encouraged to draw on the solution plan developed for that problem, and as their mathematical thinking matures, to look for generalizations that can be applied to other problem situations. The **Problem-Based Interactive Learning** activities offer students opportunities to look for regularity in the way operations behave.

For examples, see *enVisionMATH* Grade 6 Lessons 2-8, 3-10, 5-1, 6-4, 11-9, 13-1, 13-5, 15-4, 16-1, 16-2, 17-2, 17-6

Correlation of Standards for Mathematical Content

enVisionMATH® Grade 6

The following shows the alignment of *enVisionMATH* Grade 6 ©2009/2011 to the Grade 6 Common Core State Standards for High School Mathematics. Included in this correlation are the supplemental lessons that will be available as part of the transitional support that Pearson is providing. These lessons will be available in the summer 2011.

Standards for Mathematical Content Grade 6		Where to find in *enVisionMATH* ©2009/2011
Ratios and Proportional Relationships		
Understand ratio concepts and use ratio reasoning to solve problems.		
6.RP.1	Understand the concept of a ratio and use ratio language to describe a ratio relationship between two quantities.	12-1, 12-6
6.RP.2	Understand the concept of a unit rate a/b associated with a ratio $a:b$ with $b \neq 0$, and use rate language in the context of a ratio relationship.	12-3, 12-6, 13-2
6.RP.3	Use ratio and rate reasoning to solve real-world and mathematical problems, e.g., by reasoning about tables of equivalent ratios, tape diagrams, double number line diagrams, or equations.	12-4, 12-6, 13-1, 13-3a
6.RP.3.a	Make tables of equivalent ratios relating quantities with whole number measurements, find missing values in the tables, and plot the pairs of values on the coordinate plane. Use tables to compare ratios.	13-1, 13-3b
6.RP.3.b	Solve unit rate problems including those involving unit pricing and constant speed.	12-4, 12-5, 13-2, 13-4
6.RP.3.c	Find a percent of a quantity as a rate per 100 (e.g., 30% of a quantity means 30/100 times the quantity); solve problems involving finding the whole, given a part and the percent.	14-5, 14-7
6.RP.3.d	Use ratio reasoning to convert measurement units; manipulate and transform units appropriately when multiplying or dividing quantities.	16-1, 16-2, 16-4, 16-5a

Standards for Mathematical Content Grade 6		Where to find in *enVisionMATH* ©2009/2011
The Number System		
Apply and extend previous understandings of multiplication and division to divide fractions by fractions.		
6.NS.1	Interpret and compute quotients of fractions, and solve word problems involving division of fractions by fractions, e.g., by using visual fraction models and equations to represent the problem.	9-1, 9-3, 9-4, 9-5
Compute fluently with multi-digit numbers and find common factors and multiplies.		
6.NS.2	Fluently divide multi-digit numbers using the standard algorithm.	3-5, 3-8, 3-5a
6.NS.3	Fluently add, subtract, multiply, and divide multi-digit decimals using the standard algorithm for each operation.	3-2, 3-4, 3-5, 3-6, 3-7, 3-8, 3-10, 8-5
6.NS.4	Find the greatest common factor of two whole numbers less than or equal to 100 and the least common multiple of two whole numbers less than or equal to 12. Use the distributive property to express a sum of two whole numbers 1–100 with a common factor as a multiple of a sum of two whole numbers with no common factor.	5-3, 5-6, 7-2
Apply and extend previous understandings of numbers to the system of rational numbers.		
6.NS.5	Understand that positive and negative numbers are used together to describe quantities having opposite directions or values (e.g., temperature above/below zero, elevation above/below sea level, credits/debits, positive/negative electric charge); use positive and negative numbers to represent quantities in real-world contexts, explaining the meaning of 0 in each situation.	10-1
6.NS.6	Understand a rational number as a point on the number line. Extend number line diagrams and coordinate axes familiar from previous grades to represent points on the line and in the plane with negative number coordinates.	10-1,10-3,10-9
6.NS.6.a	Recognize opposite signs of numbers as indicating locations on opposite sides of 0 on the number line; recognize that the opposite of the opposite of a number is the number itself, e.g., $-(-3) = 3$, and that 0 is its own opposite.	10-1, 10-2a
6.NS.6.b	Understand signs of numbers in ordered pairs as indicating locations in quadrants of the coordinate plane; recognize that when two ordered pairs differ only by signs, the locations of the points are related by reflections across one or both axes.	10-2a, 10-9
6.NS.6.c	Find and position integers and other rational numbers on a horizontal or vertical number line diagram; find and position pairs of	6-5, 10-1, 10-3, 10-9

Standards for Mathematical Content Grade 6		Where to find in *enVisionMATH* ©2009/2011
	integers and other rational numbers on a coordinate plane.	
6.NS.7	Understand ordering and absolute value of rational numbers.	10-1, 10-2, 10-3
6.NS.7.a	Interpret statements of inequality as statements about the relative position of two numbers on a number line diagram.	10-2, 10-3
6.NS.7.b	Write, interpret, and explain statements of order for rational numbers in real-world contexts.	10-2, 10-3
6.NS.7.c	Understand the absolute value of a rational number as its distance from 0 on the number line; interpret absolute value as magnitude for a positive or negative quantity in a real-world situation.	10-1, 10-2a
6.NS.7.d	Distinguish comparisons of absolute value from statements about order.	10-2a
6.NS.8	Solve real-world and mathematical problems by graphing points in all four quadrants of the coordinate plane. Include use of coordinates and absolute value to find distances between points with the same first coordinate or the same second coordinate.	10-9, 10-10, 10-10a

Standards for Mathematical Content Grade 6		Where to find in *enVisionMATH* ©2009/2011
Expressions and Equations		
Apply and extend previous understandings of arithmetic to algebraic expressions.		
6.EE.1	Write and evaluate numerical expressions involving whole-number exponents.	1-3
6.EE.2	Write, read, and evaluate expressions in which letters stand for numbers.	2-1, 2-6
6.EE.2.a	Write expressions that record operations with numbers and with letters standing for numbers. For example, express the calculation "Subtract *y* from 5" as 5 – *y*.	2-1, 2-7, 2-8
6.EE.2.b	Identify parts of an expression using mathematical terms (sum, term, product, factor, quotient, coefficient); view one or more parts of an expression as a single entity.	2-6
6.EE.2.c	Evaluate expressions at specific values of their variables. Include expressions that arise from formulas used in real-world problems. Perform arithmetic operations, including those involving whole number exponents, in the conventional order when there are no parentheses to specify a particular order (Order of Operations).	2-6, 3-8, 17-1, 17-2, 17-3, 18-2
6.EE.3	Apply the properties of operations to generate equivalent expressions.	2-2, 2-3, 2-4, 2-6, 4-1, 2-5a
6.EE.4	Identify when two expressions are equivalent (i.e., when the two expressions name the same number regardless of which value is substituted into them).	4-1
Reason about and solve one-variable equations and inequalities.		
6.EE.5	Understand solving an equation or inequality as a process of answering a question: which values from a specified set, if any, make the equation or inequality true? Use substitution to determine whether a given number in a specified set makes an equation or inequality true.	3-9a, 4-2, 4-4, 15-7, 15-6a
6.EE.6	Use variables to represent numbers and write expressions when solving a real-world or mathematical problem; understand that a variable can represent an unknown number, or, depending on the purpose at hand, any number in a specified set.	2-1, 2-6, 3-9a, 4-2, 4-5,
6.EE.7	Solve real-world and mathematical problems by writing and solving equations of the form $x + p = q$ and $px = q$ for cases in which p, q and x are all nonnegative rational numbers.	4-2, 4-3, 4-4, 4-5, 9-6, 15-1, 17-1, 17-2, 17-3
6.EE.8	Write an inequality of the form $x > c$ or $x < c$ to represent a constraint or condition in a real-world or mathematical problem. Recognize that	15-6a

Standards for Mathematical Content Grade 6		Where to find in *enVisionMATH* ©2009/2011
	inequalities of the form $x > c$ or $x < c$ have infinitely many solutions; represent solutions of such inequalities on number line diagrams.	
Represent and analyze quantitative relationships between dependent and independent variables.		
6.EE.9	Use variables to represent two quantities in a real-world problem that change in relationship to one another; write an equation to express one quantity, thought of as the dependent variable, in terms of the other quantity, thought of as the independent variable. Analyze the relationship between the dependent and independent variables using graphs and tables, and relate these to the equation.	15-2, 15-3, 15-4, 15-5

Standards for Mathematical Content Grade 6		Where to find in *enVisionMATH* ©2009/2011
Geometry		
Solve real-world and mathematical problems involving area, surface area, and volume.		
6.G.1	Find the area of right triangles, other triangles, special quadrilaterals, and polygons by composing into rectangles or decomposing into triangles and other shapes; apply these techniques in the context of solving real-world and mathematical problems.	17-2, 17-3, 17-4a
6.G.2	Find the volume of a right rectangular prism with fractional edge lengths by packing it with unit cubes of the appropriate unit fraction edge lengths, and show that the volume is the same as would be found by multiplying the edge lengths of the prism. Apply the formulas $V = l \times w \times h$ and $V = bh$ to find volumes of right rectangular prisms with fractional edge lengths in the context of solving real-world and mathematical problems.	18-4a
6.G.3	Draw polygons in the coordinate plane given coordinates for the vertices; use coordinates to find the length of a side joining points with the same first coordinate or the same second coordinate. Apply these techniques in the context of solving real-world and mathematical problems.	10-10a
6.G.4	Represent three-dimensional figures using nets made up of rectangles and triangles, and use the nets to find the surface area of these figures. Apply these techniques in the context of solving real-world and mathematical problems.	18-1, 18-2

Standards for Mathematical Content Grade 6		Where to find in *enVisionMATH* ©2009/2011
Statistics and Probability		
Develop understanding of statistical variability.		
6.SP.1	Recognize a statistical question as one that anticipates variability in the data related to the question and accounts for it in the answers.	19-4a
6.SP.2	Understand that a set of data collected to answer a statistical question has a distribution which can be described by its center, spread, and overall shape.	19-4b
6.SP.3	Recognize that a measure of center for a numerical data set summarizes all of its values with a single number, while a measure of variation describes how its values vary with a single number.	19-5, 19-6a, 19-6c
Summarize and describe distributions.		
6.SP.4	Display numerical data in plots on a number line, including dot plots, histograms, and box plots.	19-6, 19-7, 19-8a
6.SP.5	Summarize numerical data sets in relation to their context, such as by:	
6.SP.5.a	Reporting the number of observations.	19-6, 19-9b
6.SP.5.b	Describing the nature of the attribute under investigation, including how it was measured and its units of measurement.	19-4a, 19-9b
6.SP.5.c	Giving quantitative measures of center (median and/or mean) and variability (interquartile range and/or mean absolute deviation), as well as describing any overall pattern and any striking deviations from the overall pattern with reference to the context in which the data were gathered.	19-5, 19-6a, 19-6b, 19-6c, 19-9b
6.SP.5.d	Relating the choice of measures of center and variability to the shape of the data distribution and the context in which the data were gathered.	19-8, 19-10, 19-9a, 19-9b